Special Education District of Lake County

Common Core Math Curriculum Framework

(Kindergarten - grade 2)

Updated 12/5/2012

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Common Core Framework Concept Reference

Understanding the SEDOL Common Core Math Curriculum Frameworks

What makes the Common Core Standards Different from the Illinois State Standards?

The Common Core State Standards were developed through a state-led initiative that drew on the expertise of teachers, researchers and content experts from across the country. While Illinois, and many other states already had state standards, it was very clear that they lacked focus, coherence, rigor, and most importantly they did not align with College Readiness Standards. This became a problem when looking at high-stakes tests such as the PSAE/ACT. While Illinois Learning Standards aligned with the ISAT exams, they fell short when aligned to the PSAE/ACT. This is why many K-12 districts saw a dip in performance between 8th and 11th grade.

Illinois was not alone. If one compares standards from state to state, they differ tremendously. Therefore, the percentage of students meeting or exceeding individual state standards were never close to being the same. Some state's standards were more rigorous than others resulting in states with less demanding standards to have a higher rate in students meeting or exceeding standards.

For years there has been a need for greater focus in U.S. mathematics education. Math education in the U.S. was seen by many other countries as being a mile wide and an inch deep. Prior to the Common Core, state standards were making little progress in terms of organizing math so that the subject made sense. States forced teachers to cram in a wide range of math concepts with little depth and no logical progression or relationship between concepts.

The Common Core States Standards for Mathematics were designed with focus, coherence, and rigor in mind. The Standards narrow the scope of content in each grade so that students can focus more deeply on the few remaining concepts. While 'narrowing' in education is often viewed negatively because it is associated with cuts in areas like arts, technology, resources, etc. In terms of math, narrowing means "teaching less, learning more;" allowing math to "lose a few pounds and gain some muscle." With more coherence between standards, math is no longer a disjointed list of concepts, but a logical progression of principles that build within and across grade levels. To help students meet the expectations of the standards, educators will need to rigorously pursue, with equal intensity, conceptual understanding, procedural skill and fluency, and applications. Through a greater depth of understanding of less concepts The Standards strive for greater achievement at the college- and career-ready level.

While the Common Core State Standards for Mathematics are a step in the right direction in helping students achieve in mathematics, alone they cannot raise achievement. Standards do not stay up late at night working on lesson plans, or stay after school making sure every student learns - it's teachers who do that. Through correct implementation, a properly aligned curriculum, and supportive staff, administrators, and educational leaders, the Standards will become a reality in schools.

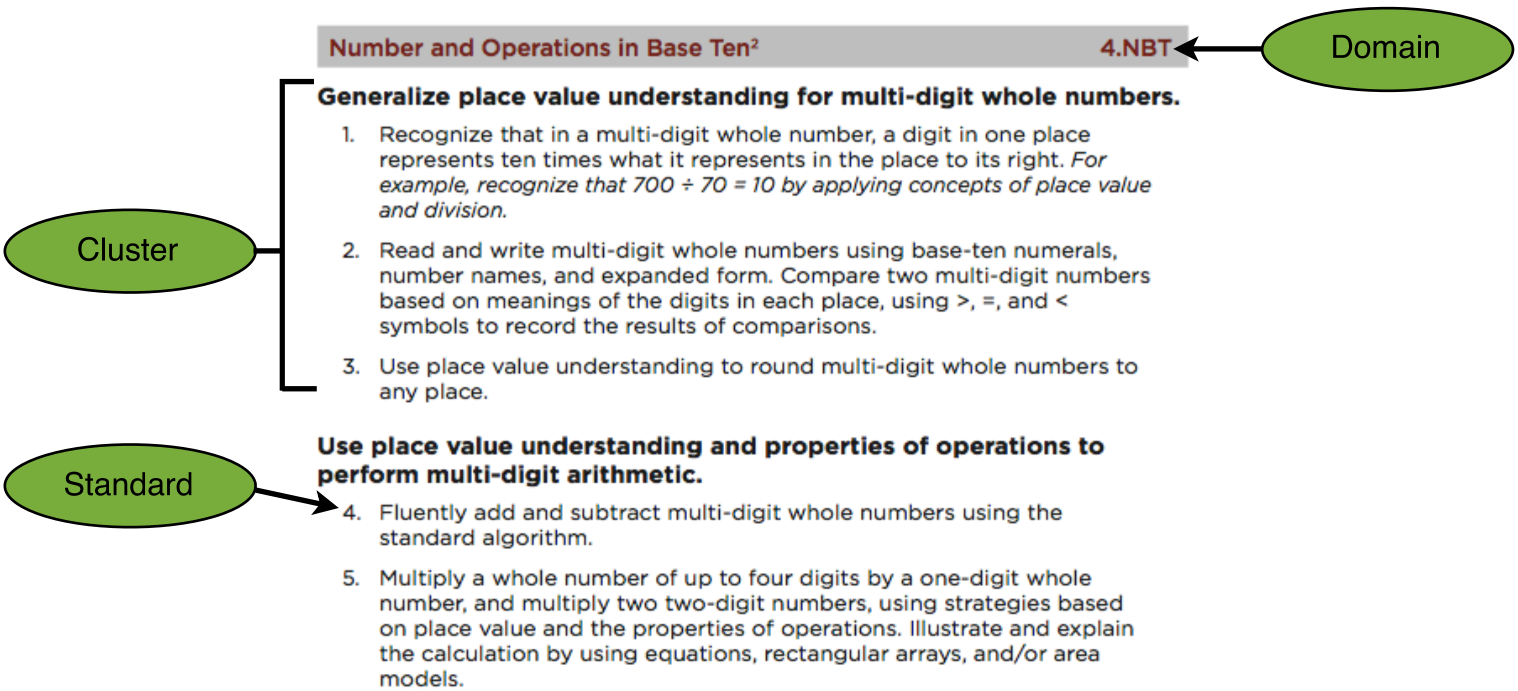
How are the Common Core Standards Structured?

The Common Core State Standards for Mathematics are broken into:

Domains - are larger groups of related standards. Standards from different domains may sometimes be closely related.

Clusters - summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Standards - define what students should understand and be able to do.



Number and Operations in Base Ten[2] **4.NBT**

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

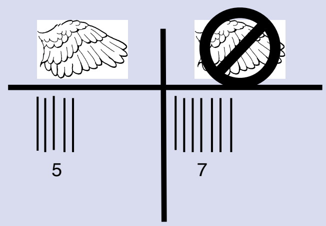
Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

How is the SEDOL Common Core Math Curriculum Framework Structured?

The SEDOL Common Core State Standards for Mathematics Curriculum Framework takes all of the Common Core State Standards that were traditionally organized by separate grade levels, and organizes them in a progressional format across grade levels. The Standards in the SEDOL Curriculum Framework are the original Common Core Standards. No wording has changed and the standards remain with their proper cluster heading and proper domain. What is different is that the standards have been organized by concept (e.g., counting) and then mapped out in such a way so that the progression of the concept can be seen across grade levels.

CCSS Domain	Measurement and Data (K.MD)	Measurement and Data (1.MD) *[illegible] will demonstrate mastery of all [illegible]evious skills and learn to:*	Measurement and Data (2.MD) *Students will demonstrate mastery of all previous skills and learn to:*
Represent and Interpret	**Classify objects and count the number of objects in each category.** 3. Classify objects into given categories; count the number of objects in each category and sort the categories by count (limit counts to less than or equal to ten).	**Represent and interpret data.** 4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or [illegible] in one category than in another.	**Represent and interpret data.** 9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. 10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put- together, take-apart, and compare problems using information presented in a bar graph.
	Instructional Strategies: [illegible] realize all the different ways objects ca[illegible]. There is not just one way to classify an object. Have them explore objects around the classroom and brainstorm all the different ways they can classify objects within the classroom[1]. Discuss with students their findings and reasonings (MP.1, MP.2, MP.3, MP.5, MP.6, MP.7). Also connect classification to other subject areas. Classification can be seen in all subject areas. For example, in reading, words can be classified based on vowel sounds, syllables, beginning letter, etc. The more students practice classification, the better they will get at determining their own measurable attributes to classify something (MP.4). After classifying and sorting objects students should count the number of objects in each category (MP.4). Then, if they have a grasp of cardinality and comparing numbers, have them talk about their finding referring to the numbers (MP.2, MP.3, MP.6, MP.8). Have students also practice with classification by guessing the rule for already sorted items (MP.1).	**Instructional Strategies:** In the previous stage, students learned how to recognize the number of objects in a group and then compared it to the number of objects in another group. Unlike the previous stage, where groups were either made for the student or specific criteria was given for sorting, students now begin to sort items by their own criteria. Students should be asked to sort a collection of items by up to three categories. Certain objects may still warrant the need for sorting categories to be given[4]. Discuss and analyze findings of sorts (MP.1, MP.3, MP.7). Reinforce previous concepts, by asking students about the number of items in each category as well as which category has more items (MP.2). Also, have students practice addition by adding up th[illegible] items (MP.1). The total number of [illegible] than or equal to 20, since additio[illegible] to 20 is the focus of this level[1]. This concept should also be conne[illegible] having students sort collections of geo[illegible]. Students should then be questioned on their sorts[1]. After students have had experience sorting objects, they may begin to create graphic representations of the sorts. They 	**Instructional Strategies:** At this stage students expand on their graphing knowledge to move from making tally counts of categories or cluster graphs to making bar graphs and picture graphs. A bar graph representing categorical data displays no additional information beyond the category counts. The bars are just a way to make the category counts easier to visually interpret. The hardest part for children is not collecting the data, but properly creating and labeling bar graphs[4]. At first students should create picture graphs where each row or bar consists of countable parts (MP.4). These graphs show items in a category and do not have a numerical scale. For example, if students were counting the different color eyes in his/her class, the graph would show eyes lined up end to end horizontally or vertically. Students would then just count how many eyes were in a row or column[1]. Also practice using scaled picture graphs where one picture may represent 2,3,4... counts which is represented in a key (MP.2, MP.5). Make sure students label the categories and the title of the graph (MP.1, MP.6). After students learn to make picture graphs, move to making bar graphs with a numerical scale (MP.4). Demonstrate how a bar graph is just like a picture graph but it has a number scale. Refer to the number scale as a count scale and discuss how it is similar to a number line diagram[4]. Practice labeling the scale on bar graphs using graph paper (MP.1, MP.6). Line plots are useful tools for collecting data because they show the number of things along a numeric scale. They are made by simply drawing a number line then placing an X above the corresponding value on the line that represents each piece of data. Line plots are essentially bar graphs with a potential bar for each value on the number line[1] (MP.1)

CCSS Domains

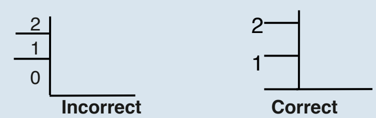
CCSS Standard

CCSS Clusters

SEDOL Instructional Strategies

Instructional alignment to 8 mathematical practices

Concept Reference

Represent and Interpret Data			
	Instructional Strategies Continued:	**Instructional Strategies Continued:** may create tables with marks for the number of items in each category. Or, they may create picture graphs in which one picture represents one object (though picture graphs are not an expectation until the next level) Students may also create cluster graphs. A cluster graph is made up of two or three labeled loops and students place the appropriate items in each loop. Objects that do not fit the criteria of any loop get placed outside of the regions. It is important that all of these forms of representation get modeled by the teacher several times before students create their own. Have students discuss and analyze representations of data (MP.1, MP.2, MP.3, MP.4, MP. 5, MP.6, MP.7, MP.8).	**Instructional Strategies Continued:** At this level students are generating a set of data on their own. For example, students might measure the length of their hand and record values to make a class data chart (MP.4, MP. 5). Students need to understand that measurement data gets represented in a line plot (MP.2). Instruct students on how to make a line plot by first making a number line that ranges from the greatest and least values and then making marks above the line for each data piece[4] (MP.4, MP.6). Discuss the similarities and differences between bar graphs, picture graphs, and line plots. Discuss how line plots allow one to see gaps between data points (e.g., four data points on 67 and one on 69, but none on 68), while in bar graphs there are no "gaps" between categories (e.g., green and blue)[4] (MP.1, MP.3, MP.6, MP. 7). Pose questions that use information in the graphs. Ask questions that involve simple put together, take-apart, and compare problems found in table 1 of the CCSS[1].
	SEDOL Common Misconceptions **Common Misconceptions:** St... hard time narrowing down the method ... in presorted objects. They may give a set of criteria that is too broad. For example, if some shapes were sorted into piles of triangles and circles, a student might say the classification of the triangle pile is "shapes" rather than triangles. Help students see that sometimes they need to look closer at the objects in a set to see if they have more specific details in common.	**Common Misconceptions:** Students may have a hard time coming up with the criteria to sort a group of object by. They may choose criteria to broad or too narrow. Or, they may choose criteria such that an object fits into two criteria making the sort difficult. Students that struggle with coming up with criteria to sort objects need to still be directed to how groups should be sorted. Some students may still need the criteria for sorting told directly to them, while other might just need a direction (e.g., look at the colors of the items).	**Common Misconceptions:** It might be natural for a student to want to represent measurement data with a bar graph, but measurement data points (e.g., 25in.) are not categories and therefore cannot be put in a bar graph. They should be represented through a line plot. Students often have trouble labeling a scale correctly on graph paper. They often write the scale numbers within the spaces on the grid rather than next to the tics where the horizontal and vertical lines meet. Stress this when instructing students on how to make bar graphs. e.g. 
	SEDOL Assessment Procedures **Assessment Procedure:** Assessment K.G. (AMC)	**Assessment Procedure:** Assessment 1.G (AMC)	**Assessment Procedure:** Assessment 2.G (AMC)
	SEDOL Vocabulary **Vocabulary:** Sort	**Vocabulary:** Number count Cluster graph Category	**Vocabulary:** Picture graph Bar graph Line plot Scale

The SEDOL Common Core Math Curriculum Framework is broken into five parts for each concept.

1. **Common Core Standard:** Taken directly from the Common Core State Standards
2. **Instructional Strategies:** An unwrapping of standards along with instructional strategies and pointers that can be used to teach the standard(s) within a given concept. References to the 8 mathematical practices (See page 12) are also given.
3. **Common Misconceptions:** A description of some common misconceptions children have when learning given standard(s) and teaching suggestions to avoid future misconceptions.
4. **Assessment Procedures:** Assessments that are currently available to test students' understanding of given standard(s).
5. **Vocabulary:** A list of vocabulary that applies to given standard(s)

How should the Common Core Math Curriculum Frameworks be utilized?

SEDOL Curriculum Frameworks were designed to allow teachers to trace how mathematical concepts are developed over time, known as a vertical alignment. Although this is known as a vertical alignment, it is presented in a horizontal fashion within the Framework. This allows teachers to map out previous concepts that a child needs to learn before conceptually understanding more difficult concepts. Quite often teachers struggle in understanding why a student is not grasping a mathematical concept. The SEDOL Common Core Curriculum Framework allows for teachers to look for gaps in student's understanding of a concept that may be impeding understanding of many mathematical concepts.

While the SEDOL Common Core Curriculum Framework organizes concepts through a vertical alignment (presented horizontally) in the order they should be taught, the framework does not dictate the order in which concepts should be taught within a particular grade level band (presented vertically). A student may not fall into one grade level for all concepts. For example, a student may have mastered standard 1.NBT.1[1] but still be working on standard K.OA.3[2]. In addition, many concepts within one column may be taught simultaneously. For example, students can be working on standard 1.OA.2[3] while working on standard 1.MD.4[4].

It should also be noted, that not all the standards should be equally weighted in importance. Some clusters require greater emphasis than others based on the depth of ideas, the time that they take to master, their importance to future mathematics, and/or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice (see page 12).

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

[1] Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral

[2] (Decompose numbers less than or equal to 10 into pairs in more than one way, *e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1)*)

[3] (Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, *e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.)*

[4] (Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.)

Where is there more emphasis in each grade level?

In the table below is a breakdown of the quantity of standards found within each Common Core domain. This table allows for one to see the number of standards in each grade, what domains may be of larger focus within each grade, and what domains may require more instructional time to meet all of the standards. For example, in kindergarten and first grade measurement and data have a small part in the overall curriculum, while in second grade it has a much larger part in the curriculum. This does not mean it is of more importance than the other domains, it just means that there are more standards on measurement and data to meet within second grade.

Common Core Domain	Kindergarten	First Grade	Second Grade
Counting and Cardinality (CC)	9	0	0
Number and Operations in Base Ten (NBT)	1	8	10
Operations and Algebraic Thinking (OA)	5	8	4
Geometry (G)	6	3	3
Measurement and Data (MD)	3	4	10
Total	24	23	27

Below is another way to view the above standard breakdown based on the concept references within SEDOL's Common Core Framework. It provides a more specific breakdown of what concepts may or may not play a larger role in the curriculum.

SEDOL Framework Concept Reference	Kindergarten	First Grade	Second Grade
Counting (CC/NBT)	7	1	2
Order and Compare Numbers (CC/NBT)	2	1	1
Addition and Subtraction (OA)	5	4	2
Multiplication and Division (OA)	0	0	2
Place Value (NBT)	1	6	6
Operation Comprehension (OA)	0	2	1
Algebraic Thinking (OA)	0	2	0
Measurement (MD)	2	2	6
Time and Money (MD)	0	1	2
Represent and Interpret Data (MD)	1	1	2
Shape Manipulation and Spatial Reasoning (S)	2	2	2
Shape Recognition (S)	4	1	1
Total	24	23	27

How do I know what standards I should emphasize throughout the year?

The SEDOL Common Core Math Curriculum Framework identifies which clusters are major, supporting, and additional through a color-coded star (★) next to each cluster.

★ Major Cluster: Should make up about two-thirds to three-quarters of the math curriculum. Should especially predominate in the first half of the year.

★ Supporting Cluster: Should make up about one-third to one-fourth of the math curriculum. These are likely taught in conjunction with a major cluster.

☆ Additional Cluster: Should make up a small fraction of the math curriculum. These are often taught in isolation, but can be taught in conjunction with a major cluster.

The SEDOL Common Core Math Curriculum Framework should be utilized along with the SEDOL Common Core Math Curriculum Scope and Sequence and Alignment documents. The Scope and Sequence documents will help in breaking down standards throughout the year to allow for a logical progression. The alignment document will identify what curricular tools are available to teach any particular Common Core standard.

What are the required computational fluencies in each grade?

Fluency is defined as being efficient, accurate, and flexible in one's thinking. This means that students should not only be able to provide correct answers quickly but also to use facts and computation strategies they know to efficiently determine answers that they do not know. Therefore when it is stated that students should be fluent in adding/subtracting within 1,000,000, this does not mean they have memorized all facts within 1,000,000 but rather that they have developed efficient, accurate, and flexible ways to solve addition/subtraction problems within 1,000,000.

Grade	Standard	Required Fluency
K	K.OA.5	Add/Subtract within 5
1	1.OA.6	Add/subtract within 10
2	2.OA.2 2.NBT.5	Add/Subtract within 20 Add/Subtract within 100
3	3.OA.7 3.NBT.2	Multiply/divide within 100 Add/subtract within 1000
4	4.NBT.4	Add/subtract within 1,000,000
5	5.NBT.5	Multi-digit multiplication
6	6.NS.2 6.NS.3	Multi-digit division Multi-digit decimal operations

Progress to Algebra in Grades k-8

K	1	2	3	4	5	6	7	8	High School
Counting and Cardinality Know number names and the count sequence Count to tell the number of objects Compare numbers									**Number and Quantity**
Number and Operations in Base Ten Work with numbers 11-19 to gain foundations for place value	**Number and Operations in Base Ten** Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract	**Number and Operations in Base Ten** Understand place value Use place value understandings and properties of operations to add and subtract	**Number and Operations in Base Ten** Use place value understanding and properties of operations to perform multi-digit arithmetic.	**Number and Operations in Base Ten** Generalize place value understanding for multi-digit whole numbers. Use place value understanding and properties of operations to perform multi-digit arithmetic.	**Number and Operations in Base Ten** Understand the place value system. Perform operations with multi-digit whole numbers and with decimals to hundredths.	**Ratios and Proportional Relationships** Understand ratio concepts and use ratio reasoning to solve problems.	**Ratios and Proportional Relationships** Analyze proportional relationships and use them to solve real-world and mathematical problems.		
			Number and Operations - Fractions Develop understanding of fractions as numbers.	**Number and Operations - Fractions** Extend understanding of fraction equivalence and ordering. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. Understand decimal notation for fractions, and compare decimal fractions.	**Number and Operations - Fractions** Use equivalent fractions as a strategy to add and subtract fractions. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	**The Number System** Apply and extend previous understandings of multiplication and division to divide fractions by fractions. Compute fluently with multi-digit numbers and find common factors and multiples. Apply and extend previous understandings of numbers to the system of rational numbers.	**The Number System** Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	**The Number System** Know that there are numbers that are not rational, and approximate them by rational numbers.	
Operations and Algebraic Thinking Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	**Operations and Algebraic Thinking** Add and subtract within 20 Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction	**Operations and Algebraic Thinking** Represent and solve problems involving addition and subtraction Add and subtract within 20 Work with groups of objects to gain foundations for multiplication Use place value understanding and properties of operations to add and subtract. Work with addition and subtraction equations	**Operations and Algebraic Thinking** Represent and solve problems involving multiplication and division. Understand properties of multiplication and the relationship between multiplication and division.	**Operations and Algebraic Thinking** Use the four operations with whole numbers to solve problems. Gain familiarity with factors and multiples. Multiply and divide within 100. Solve problems involving the four operations, and identify and explain patterns in arithmetic.	**Operations and Algebraic Thinking** Write and interpret numerical expressions. Analyze patterns and relationships. Generate and analyze patterns.	**Expressions and Equations** Apply and extend previous understandings of arithmetic to algebraic expressions. Reason about and solve one-variable equations and inequalities. Represent and analyze quantitative relationships between dependent and independent variables.	**Expressions and Equations** Use properties of operations to generate equivalent expressions. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	**Expressions and Equations** Work with radicals and integer exponents. Understand the connections between proportional relationships, lines, and linear equations. Analyze and solve linear equations and pairs of simultaneous linear equations.	**Algebra and Functions**
								Functions Define, evaluate, and compare functions. Use functions to model relationships between quantities.	
Geometry Analyze, compare, create and compose shapes Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)	**Geometry** Reason with shapes and their attributes	**Geometry** Reason with shapes and their attributes	**Geometry** Reason with shapes and their attributes.	**Geometry** Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	**Geometry** Graph points on the coordinate plane to solve real-world and mathematical problems. Classify two-dimensional figures into categories based on their properties.	**Geometry** Solve real-world and mathematical problems involving area, surface area, and volume.	**Geometry** Draw, construct, and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	**Geometry** Understand congruence and similarity using physical models, transparencies, or geometry software. Understand and apply the Pythagorean Theorem. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	**Geometry**
Measurement and Data Describe and compare measurable attributes Classify objects and count the number of objects in each category	**Measurement and Data** Measure lengths indirectly and by iterating length units Tell and write time Represent and interpret data	**Measurement and Data** Measure and estimate lengths in standard units Relate addition and subtraction to length Work with time and money Represent and interpret data	**Measurement and Data** Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. Represent and interpret data. Geometric measurement: understand concepts of area and relate area to multiplication and to addition. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	**Measurement and Data** Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. Represent and interpret data. Geometric measurement: understand concepts of angle and measure angles.	**Measurement and Data** Convert like measurement units within a given measurement system. Represent and interpret data. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	**Statistics and Probability** Develop understanding of statistical variability. Summarize and describe distributions.	**Statistics and Probability** Use random sampling to draw inferences about a population. Draw informal comparative inferences about two populations. Investigate chance processes and develop, use, and evaluate probability models.	**Statistics and Probability** Investigate patterns of association in bivariate data.	**Statistics and Probability**

Addition and Subtraction Situations (by grade level)

	Results Unknown	Change Unknown	Start Unknown
Add to	**A** bunnies sat on the grass. **B** more bunnies hopped there. How many bunnies are on the grass now? K Mastery **A + B =**□	**A** bunnies were sitting on the grass. Some more bunnies hopped there. Then there were **C** bunnies. How many bunnies hopped over to the first A bunnies? 1st Mastery **A +** □ **= C**	Some bunnies were sitting on the grass. **B** more bunnies hopped there. Then there were **C** bunnies. How many bunnies were on the grass before? 2nd Mastery □**+ B = C**
Take from	**C** apples were on the table. I ate **B** apples. How many apples are on the table now? K Mastery **C – B =** □	**C** apples were on the table. I ate some apples. Then there were **A** apples. How many apples did I eat? 1st Mastery **C –** □ **= A**	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? 2nd Mastery □**– B = A**
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**
Put Together or Take Apart	**A** red apples and **B** green apples are on the table. How many apples are on the table? K Mastery **A + B =** □	**C** apples are on the table. **A** are red and the rest are green. How many apples are green? 1st Mastery **A +** □ **= C** **C – A =** □	Grandma has **C** flowers. How many can she put in her red vase and how many in her blue vase? 1st Mastery **C =** □ **+** □
Compare	"How many more?" version. Lucy has **A** apples. Julie has **C** apples. How many more apples does Julie have than Lucy? 1st Mastery	"More" version suggests operation. Julie has **B** more apples than Lucy. Lucy has **A** apples. How many apples does Julie have? 1st Mastery	"Fewer" version suggests operation. Lucy has **B** fewer apples than Julie. Julie has **C** apples. How many apples does Lucy have? 1st Mastery
	"How many fewer?" version. Lucy has **A** apples. Julie has **C** apples. How many fewer apples does Lucy have than Julie? 1st Mastery **A +** □ **= C** **C – A =** □	"Fewer" version suggests wrong operation. Lucy has **B** fewer apples than Julie. Lucy has **A** apples. How many apples does Julie have? 2nd Mastery **A + B =**□	"More" version suggests wrong operation. Julie has **B** more apples than Lucy. Julie has **C** apples. How many apples does Lucy have? 2nd Mastery **C – B =** □ □**+ B = C**

Darker shading indicates the three Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Un-shaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2.

What are the 8 Mathematical Practices?

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. They are not intended to be taught in isolation. They are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, and apply the mathematics to practical situations. They may not use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

See the table below for a further explanation of the 8 Mathematical Practices.

Mathematical Practices	Student
1. Make sense of problems and persevere in solving them.	• Discuss problem solving methods. • Check answer to see if it "makes sense". • Use manipulatives to help them understand and solve a problem. • Create a method for solving a puzzle or problem. • Recognize patterns or relationships in problems. • Demonstrate understanding of math foundation before developing algorithm.
2. Reason abstractly and quantitatively.	• Break down problems into symbols. • Understand symbols in a problem • Represent quantities and problems in multiple ways. • Use appropriate units of measurement (people, GPA, cars, inches, pounds, etc.)
3. Construct viable arguments and critique the reasoning of others.	• Use objects, drawings, diagrams, and actions to solve a problem • Demonstrate knowledge of when to utilize certain mathematical concepts and theories. • Justify their answers and explanations of how they solved the problem. • Compare the effectiveness of two possible solutions. • Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. Model with mathematics.	• Apply math skills to solve problems arising in everyday life, society, and the workplace. • Write an addition equation to describe a situation. • Apply math knowledge to solve mathematical problems. • Identify important quantities in a practical situation. • Map math relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. • Analyze math relationships to draw conclusions. • Determine if results make sense and correcting the problem solving process if it does not. • Represent with words or pictures the path to the solution of a problem. • Draw diagrams in which they represent the problem situations and relevant concepts using bars. • Understand the concepts and work out a strategy for finding the answer.
5. Use appropriate tools strategically.	• Select appropriate tools when solving a problem • Identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. • Use technological tools to explore and deepen their understanding of concepts. • Analyze graphs of functions and solutions generated using a graphing calculator. • Detect possible errors by strategically using estimation and other mathematical knowledge. • Use technology to help them visualize the results of varying assumptions, explore consequences, and compare predictions with data.

Mathematical Practices	Student
6. Attend to precision.	• Use clear definitions in discussion with others and in their own reasoning. • Communicate with mathematical terms. • Know the meaning of the symbols they choose. • Are careful about specifying units of measure, and labeling axis to clarify the correspondence with quantities in a problem. • Calculate accurately and efficiently.
7. Look for and make use of structure.	• Identify a pattern or structure (See 7×8 equals $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property) • Sort a collection of shapes according to how many sides the shapes have. • Recognize the significance of an existing line in a geometric figure. • Can use the strategy of drawing an auxiliary line for solving problems. • Step back for an overview and shift perspective. • See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. • Allow symbols or pictures to take the place of numbers to explain a concept.
8. Look for and express regularity in repeated reasoning.	• Notice if calculations are repeated. • Look both for general methods and for shortcuts. • Maintain oversight of the process, while attending to the details. • Continually evaluate the reasonableness of their results throughout the problem solving process. • Describe the pattern they discover.

Instructional Planning for the Math Block

SEDOL demonstrates commitment to high quality mathematics instruction by designating 60 - 90 minute blocks of time for mathematics instruction in elementary school classrooms. A high quality math block should include daily number sense and computational development for all students. The math block should also include regular opportunities to work in-depth with algebra, geometry, measurement and data. (Also see Three part Problem- Based Instructional Model)

Essential Components of a Math Block	What it looks like	Rationale
Number Talk (daily) Whole class (also appropriate for small group if class need requires) Kindergarten (5 min.) Grades 1-5 (5-15 min.)	● Brief time for students to develop mental computation strategies ● Teacher poses a problem- students solve it in their own way and then share their strategy with the group.	It is important for students to be flexible in their strategies, depending on the context of the problem. By seeing others' thinking, their collection of possible strategies increases.
Small Group (3-5 times per week) Grades K-5 (5-15 min.)	● Provide experiences that result in new learning for students. ● Appropriate levels of support are provided, leading ultimately to independence. ● Intervention with concepts not necessary for the whole group.	Students have a variety of needs within a classroom. Small group time allows the teacher to bring students together to build on a strength in order to come to new learning.
Independent Work Time (3-5 times per week) Grades K-5 (30-60 min.)	● Stations can be introduced in whole group or small group, with support released to independence ● Differentiated work on the concept being developed ● Stations usually related to number sense and computation. ● Teacher plays an active role setting a purpose, roaming, monitoring and interacting with students at the stations through skillful/guided questioning.	"Students need opportunities to reflect on, or create new ideas through problem-based tasks." (Van de Walle, 2004) Number concepts tend to be the most complex and require the most time to develop. This time has the greatest potential for students to get the practice they need to internalize the concept. Students first learn how to do the task and then they learn from the task.
Whole Group Lesson (1-3 times per week) The following are guidelines based on a regular education setting: Grades K-1 (30-40 min.) Grade 2 (40-50 min.) Grades 3-5 (50-60 min.)	● **Three-part, (launch, explore, summary) problem-based lesson format is recommended** ● Supports students in developing grade level appropriate concepts. ● Review of previous concepts, introduction of new concepts and/or connection of past and new learning. ● Students have time to make sense of the concept in their own way.	"Much more learning occurs and much more assessment information is available when a class works on a single problem and engages in discourse about the validity of the solution." (Van de Walle, 2004)
Assessment Opportunities (daily)	● Teachers gather assessment data during all components of the math block. ● Teachers use formal interviews, questions, observation and student work. ● Teachers use the suggested assessments listed on the SEDOL Math Assessment Plan.	"Assessment can and should happen every day as an integral part of instruction. If you restrict your view of assessment to tests and quizzes you will miss seeing how assessment can help students grow. (Van de Walle, 2004)

PROBLEM-BASED THREE PART LESSON INSTRUCTIONAL MODEL

Problem-centered teaching opens the mathematics classroom to exploring, conjecturing, reasoning, and communication. This model is very different from the “transmission” model in which teachers tell students facts and demonstrate procedures and then students memorize the facts and practice the procedures. This model looks at instruction in three phases: launching, explore, and summary.

Launch

In the first phase, the teacher launches the problem with the whole class. This involves helping students understand the problem setting, the mathematical context, and the challenge. The following questions can help the teacher prepare for the launch:

- What are students expected to do?
- What do the students need to know to understand the context of story and the challenge of the problem?
- What difficulties can I foresee for students?
- How can I keep from giving away too much of the problem?

The launch phase is also the time when the teacher introduces new ideas, clarifies definitions, reviews old concepts, and connects the problem to past experiences of the student. It is critical that, while giving students a clear picture of what is expected, the teacher leaves the potential of the task intact. He or she must be careful not to tell too much and lower the challenge of the task to something routine or to cut off the rich array of strategies that may evolve from an open launch of the problem.

Adapted from Aurora Public School Curriculum Guide 2008

Explore

In the explore phase, students work individually, in pairs, in small groups, or occasionally as a whole class to solve the problem. As they work, they gather data, share ideas, look for patterns, make conjectures, and develop problem-solving strategies. It is inevitable that students will exhibit variation in their progress. The teacher's role during this phase is to move about the classroom, to observe individual performance, and to select specific student work samples to be shared during the summary phase. The teacher helps students persevere in their work and differentiate their work by asking appropriate questions and providing confirmation and redirection where needed. For students who are interested in and capable of deeper investigation, the teacher may provide additional challenges related to the problem. Although it is imperative that all students be given enough time and opportunity to thoroughly work on the problem, it is not always necessary for every student to finish the problem at this time.

The following questions can help the teacher prepare for the explore phase:

- How will I organize the students to explore this problem? (Individuals? Pairs? Groups? Whole class?)
- What materials will students need?
- How should students record and report their work?
- What different strategies can I anticipate they might use?
- What questions can I ask to encourage student conversations, thinking, and learning?
- What questions can I ask to focus their thinking if they become frustrated?
- What questions can I ask to challenge students if the initial question is "answered"?

Adapted from Aurora Public School Curriculum Guide 2008

Summary

The summary phase of instruction begins when students have gathered sufficient data or made sufficient progress toward solving the problem. In this phase, students discuss their solutions as well as the strategies they used to approach the problem, organize the data, and find the solution. During the discussion, the teacher helps students enhance their understanding of the mathematics in the problem and guide them in refining their strategies into efficient, effective problem-solving techniques.

Although the summary discussion is led by the teacher who has collected specific student work samples he or she would like shared, students play a significant role. Ideally, they should pose conjectures, question each other, offer alternatives, provide reasons, refine their strategies and conjectures and make connections. As a result of the discussion, students should become more skillful at using the ideas and techniques that come out of the experience with the problem.

During the summary phase, content goals of the problem, investigation, and unit can be addressed, allowing the teacher to assess the degree to which students are developing their mathematical knowledge. At this time, teachers can make additional instructional decisions that will enable all students to reach the mathematical goals of the activities. The following questions can help the teacher prepare for the summary:

- How can I help the students make sense of and appreciate the variety of methods that may be used?
- How can I orchestrate the discussion by choosing specific student work samples that will help students summarize their thinking about the problems?
- What concepts or strategies need to be emphasized?
- What ideas do *not* need closure at this time?
- What definitions or strategies do we need to generalize?
- What connections and extensions can be made?
- What new questions might arise and how do I handle them?
- What will I do to follow-up, practice, or apply the ideas after the summary?

Adapted from Aurora Public School Curriculum Guide 2008

NUMBER TALKS

What is a number talk?

Number talks are a regular opportunity for students to engage in reasoning and meaning making in order to increase computational fluency. Students publicly communicate their thinking and develop flexible strategies. Strategies are scripted using a variety of models.

Why do a number talk?

- Number talks provide an opportunity for students to build on and make connections to prior understandings.
- Students have the opportunity to develop flexibility and fluency with mathematics.
- Teachers gather formative data about students' understanding.
- Number talks establish and reinforce a culture for math discourse leading to students' independence and agency.
- Number talks provide a visual record of student thinking that can be accessed in the classroom.

What does a number talk look like?

- The teacher poses a problem or a series of related problems to the whole class or a small group.
- The teacher provides adequate time for students to mentally compute a solution.
- The teacher accepts and records all answers.
- The teacher asks for students to explain their answers and scripts multiple student responses.
- Students have the opportunity to question, clarify and self-correct.
- The group comes to agreement on a correct solution.
- The pacing is quick to hold student engagement; over time all strategies are explored, but every student will not be heard every day.

How do teachers support number talks?

- Teachers provide manipulatives or models consistently to help students understand the underlying structure of numbers.
- Teachers choose problems based on student strengths. Problems can be posed in a context to support students making meaning.
- Teachers facilitate student talk by limiting their voice to clarification questions.
- Teacher's mathematical notation of solutions puts students' thinking into a visual format or model.
- Teachers maintain a neutral attitude while accepting and scripting solutions.
- While mathematically notating, teachers use a variety of visual models to represent student thinking.
- Mathematically notated student responses are posted as a classroom visual support.
- Teachers and students notice and label student strategies that can be added to the classroom record.
- Teachers choose problems based on both their observations of student work and grade level goals.
- Teachers choose problems and/or models that allow all students access to the problem.

What behaviors to look for?

- Students are listening to each other.
- Students show increased confidence, flexibility, and fluency with number and number sense.
- All students actively engage in trying the problem.
- A variety of strategies are suggested.
- Many student voices are heard and validated.

What are some professional resources that support number talks?

- Van De Walle, Teaching Developmentally.
- Richardson, Kathy, Planning Guide for Developing Number Sense
- Richardson, Kathy, Math Time: Thinking with Numbers video

CCSS Domain	Counting and Cardinality (K.CC)	Number and Operations in Base Ten (1.NBT) *Students will demonstrate mastery of all previous skills and learn to:*	Number and Operations in Base Ten (2.NBT) *Students will demonstrate mastery of all previous skills and learn to:*
Counting	**Know number names and the count sequence.** 1. Count to 100 by ones and by tens. 2. Count forward beginning from a given number within the known sequence 3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects) **Count to tell the number of objects.** 4. Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. 5. Count to answer "how many?" questions about as many as 20 arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.	**Extend the counting sequence.** 1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	**Understand place value.** 2. Count within 1000; skip-count by 5s, 10s, and 100s. 3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
	Instructional Strategies: At this stage it is imperative that students are provided with opportunities to connect and discuss mathematical language and symbols to experiences in their everyday lives. By providing students with support to see patterns and connections in the world around them they are gaining the foundation to make meaning and mathematize the real world[1] (MP.4). Be sure to involve students in mathematical discussions involving questions that give insight on the student's thought process, such as, "How did you get that?" and "Why is that true?" (MP.3, MP.6) For many children at this stage counting may be a rote activity for awhile. This is not bad. Saying numbers as a rote procedure is key to laying the foundation for conceptualizing the idea of counting. Students should know how to count before they understand cardinality[2]. Once students have begun to understand counting, it is important that they start to understand cardinality of numbers and objects. At this level, counting objects in a line is the	**Instructional Strategies:** In the previous stage, students learned to count to 20 and they learned about the cardinality of numbers. Now, students expand on their knowledge of numbers and cardinality and they learn to count to 120, starting at any number less than 120. By the end of this stage students should be recognizing groups of two, three, and four without counting. They also learn to name and write all the numerals up to 120. Students should start off learning the process by counting out manipulatives and then connecting the manipulatives to a number word and a written numeral[1] (MP.1, MP.2, MP.4, MP.5). Manipulatives are key in helping students conceptualize math concepts. In helping students understand concepts, teachers often take away manipulatives too soon. Teachers often jump from working with manipulatives to having children write and "see" symbols, skipping the connecting stage in which students are to connect manipulatives to written symbols. It is important to not abandon manipulatives in mathematics too soon. Children often appear to know and understand more than they do, so do not assume that children no longer need manipulatives.	**Instructional Strategies:** The best way for children to master their counting is by allowing them opportunities to apply counting throughout the day. Students can practice skip counting through games and/or money (nickels, dimes, and dollars). Allowing them to use nickels, dimes, and dollars to practice counting by 5s, 10s, and 100s, will not only help in teaching skip counting, but also in teaching the value of certain U.S. currencies. Pictures of coins and bills can be attached to models students are familiar with. For example, a dime on a tens-frame with 10 dots or a dollar on a hundreds-frame with 100 dots[1]. This will help students make the connection between counting, numeral values, and place value (MP.2, MP.4, MP.5, MP.7). Skip counting is essential to learn because it is a skill needed to learn multiplication[3]. With repeated practice students will discover the repetitive patterns in counting (MP.8).

Counting

Instructional Strategies Continued: easiest way to help children conceptualize the cardinality of numbers. However, before they progress to the next stage of counting, they should be able to count objects in more difficult arrangements (circles, rectangular array, scattered, etc.)[1] (MP.1, MP.2).

Providing students with experiences that promote counting is key to this development. Experiences such as board games and incorporating counting into daily activities will encourage the development of cardinality (MP.2, MP.5).

As children experience counting and the one-to-one correspondence of numbers their ability to count groups will quicken; especially of small groups. Some students will learn to recognize the cardinality of a small group without even counting. This later develops into being able to recognize sub-collections within a larger group and using their cardinalities to find the cardinality of the entire collection[2] (MP.7). This may not be fully developed by the end of this stage (MP.6).

When children have mastered counting and the cardinality of numbers, they progress towards knowing how to count forward from a given number. This is a prerequisite for counting on and later addition and subtraction (MP.7, MP.8). If a student does not understand this concept, when another object has been added to a set or when given a specific number to start counting from, they will recount the set entirely. This may mean that a child does not have full understanding of cardinality and instructional focus needs to be on cardinality and not on counting forward from a given number[2].

Just like counting to 100 is initially a rote activity, writing the numbers 1-20 will initially be a rote process. Over time, students will begin to understand that the symbols they have been copying actually signify the meaning of counting (MP.2, MP.4, MP.7). Practice count words and written numerals paired with pictures within the context of life experiences to help students understand the meaning of written numbers[1].

Students should have a good understanding of counting to 100 and the cardinality of numbers before they can show success in counting out a given number of objects (K.CC.5). This is more difficult for students because they need to be fluent in counting and have enough attention to remember the number of objects being counted out (MP.7, MP.8).

Common Misconceptions: Before a student can understand the concept of counting on it is important that they understand that the last number name said in a counting sequence tells the number of objects counted (K.CC.4b). If they do not understand this, when a student is asked "How many _____ are there?" they may regard the counting performance itself as the answer to the question instead of the cardinality and therefore recount the whole set again[2]. If a student does not understand cardinality, they may also separate a set into two groups, but then re-count from the

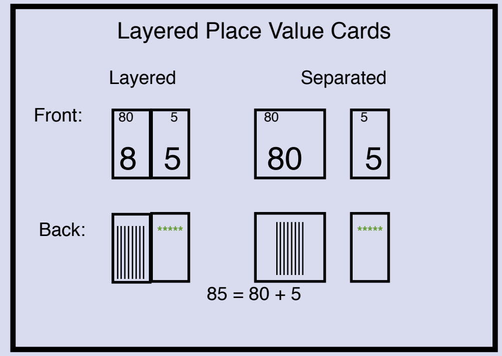
Instructional Strategies Continued: Using groupable and pregrouped (making sure students are not just attaching a name to the item) manipulatives have students work on creating models for numbers 21-120. This establishes background for the understanding of place value of numbers beyond a one in the tens digit[1] (MP.1, MP.6).

While listening to students count, focus on their transition from decade to decade and decade to century. This transition is signaled by the nine. Students should understand that the nine is the signal to generate the next decade of numbers. Students need to listen to their rhythm and pattern as they count so that they develop a strong sense of counting and cardinality[1] (MP.6, MP.7).

To help children grasp all the numbers up to 120, have a numeral list in the classroom that lists numbers 0-120 in columns of ten. This will not only help students learn the numbers 0-120, but also recognize the base-ten structure of numbers[2] (MP.4, MP.7, MP.8). Also have a hundreds chart with the numbers 1-100 and a second hundreds chart with the numbers 101-120 written in the same pattern as in the hundreds chart. Have children discuss the patterns seen within and between the two hundreds charts[1] (MP.3, MP.4, MP.5, MP.7, MP.8).

Common Misconceptions:

Instructional Strategies Continued: To help children learn how to write numbers to 1000 using base-ten numerals, number names, and expanded form utilize representations and manipulatives such as math drawings and layered place value cards (MP.1, MP.2, MP.4, MP.5). These methods will afford connections between written three-digit numbers and hundreds, tens, and ones.



Also, saying numbers out loud in terms of their base-ten units will help children understand the connection between a number's place values and its written numeral. For example, 354 is "Three-hundred fifty-four" and "three hundreds five tens four ones."[3]

Have students answer questions related to numbers to 1000 that foster mathematical thinking and reasoning. Have them explain how they got particular representations (base-ten, expanded) for a number (MP.3, MP.6).

Common Misconceptions: Some students may not move beyond thinking of numbers such as 436 as 400 ones plus 30 ones plus 6 ones. They have a hard time seeing it as 4 bundles of 10 tens (or hundreds), 3 bundles of 10 singles (or tens), and 6 singles (ones). They need to learn that 10 ones (singles) make a ten and 10 tens make a hundred. Use manipulatives to model how singles and tens can be bundled to make tens and hundreds. It is important that students connect a group of 10 ones with the word *ten* and 10 tens with the word *hundred*.

Counting	**Common Misconceptions Continued:** beginning to determine the number of objects in a set[1]. Sometimes students think that the count words used to tag an item is permanently embedded in that item. Therefore, when asked to count the objects again in a different order, they count according to how the items were “tagged” the previous time. For example, say a student counted four different colored crayons: red, blue, yellow, and green with the count words one, two, three, four. Then when asked to count the same set with the crayons arranged as blue, green, yellow, and red they may say two, four, three, one[1]. Some students might not see zero as a number. Ask students to write 0 and say *zero* to represent the number of items left when all items have been taken away. Avoid using the word *none* to represent this situation[2].	**Common Misconceptions Continued:**	**Common Misconceptions Continued:** Some students have a hard time utilizing money for skip counting. Students may have a hard time seeing a single coin or dollar as representing a value such as 5, 10, or 100. To help students see coins as representing a value other than 1, show coins on ten-frames (or five-frames) with dots that correspond to the coins value. For example, a nickel on a frame with five dots.
	Assessment Procedure: Assessment 1: Counting Objects • Task 1: Counting a Pile (K.CC.4a, K.CC.4b, K.CC.5) • Task 2: Making a Pile (K.CC.4a, K.CC.4b, K.CC.5) • Task 3: One More/One Less - In Sequence (K.CC.4c) • Task 4: One More/One Less - Not in Sequence (K.CC.4c) • Extension - One More/One Less to 100 (K.CC.1) Assessment 2: Changing Numbers (K.CC.3)	**Assessment Procedure:** **To be developed**	**Assessment Procedure:** **To be developed**
	Vocabulary: Count Numbers zero - hundred	**Vocabulary:** Numbers zero - one hundred twenty	**Vocabulary:** Numbers zero - one thousand Skip count Number Names Standard form Expanded form

CCSS Domain	Counting and Cardinality (K.CC)	Number and Operations in Base Ten (1.NBT) *Students will demonstrate mastery of all previous skills and learn to:*	Number and Operations in Base Ten (2.NBT) *Students will demonstrate mastery of all previous skills and learn to:*
Order and Compare Numbers	**Compare numbers.** 6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, *e.g., by using matching and counting strategies* (Include groups up to 10 numbers). 7. Compare two numbers between 1 and 10 presented as written numerals.	**Understand Place Value** 3. Compare two two-digit numbers based on meanings of tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**Understand Place Value** 4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
	Instructional Strategies: At this stage students compare sets by identifying which group has more than, fewer than, or the same amount as the other. Students that do not understand cardinality of numbers will begin comparing by matching objects between two sets to see if there are any "extra" in a set[2] (MP.5). Manipulatives that show different representations of numbers (dot cards, picture cards, dominoes, etc.) become tools for students to begin to realize the relationships of numbers (MP.1, MP.4, MP.5). They begin to realize the concept of more, less, and equal. Students need to understand that equal means "is the same as."	**Instructional Strategies:** Compare problems advance from the previous stage in that they no longer are just asking "Which is more/less?" and now are asking "How many more/less?". This involves a much higher level of thinking because it is essentially asking students to find a difference value without explicitly saying, "Find the difference between these two sets."[2] Help students compare numbers by at first using pictures and representations for the numbers being compared. Have the students find the "extra" in one of the sets. Make them notice that if you added the amount that was "extra" in the larger set onto the smaller set, the sets will be equal or the same. This provides a good background to addition and subtraction[2] (MP.1, MP.2, MP.4). Students will also use their knowledge of cardinality to compare numbers with the symbols <, >, and =. To help them remember the direction the comparison sign should face, remind them that the wide part of the symbol is next to the larger number[3] (MP.7). When comparing numbers point out to students that the digit	**Instructional Strategies:** When comparing numbers students need to first understand that 1 ten is greater than any amount of ones represented by a one-digit number. Then, when working with three-digit numbers, they need to understand that 1 hundred (the smallest three-digit number) is greater than any amount of tens and ones in a two-digit number. From this, students will understand why when comparing numbers with three-digits, they should first look at the hundreds place (MP.2, MP.5, MP.7). To have students practice comparing three-digit numbers, have students rearrange three numerals such as 2, 3, 6, into all the different three-digit number combinations possible (e.g., 236, 263, 326, 362, 623, 632) (MP.4). Then have them compare the numerals in the hundreds place (MP.1). Discuss students findings and encourage students to come up with a conclusion (MP.3, MP.6). Students should conclude that the numbers with the largest valued digit in the hundreds place are the biggest. They should also conclude that when two numbers have the same digit in the hundreds place, students need to compare their digits in the tens place to determine which number is bigger. A similar process would need to be

Order and Compare Numbers

Instructional Strategies Continued: It is important that discussions over pairs of numbers take place often. Questions such as “Are more children buying or bringing lunch?” when referring to a daily lunch count chart are excellent opportunities to reinforce comparing numbers (MP. 3, MP.6). Using anchors numbers of 5 and 10 are helpful in having students understand the relationship between numbers[1] (MP.2).

Once students have developed cardinality with counting, they will begin to compare sets through counting. The highest level of understanding is achieved when students can compare and explain the relationship between two *written* numbers without counting (e.g., 5 and 9) (MP.7). This means they fluently understand cardinality[2] (MP.2).

It is important that students understand that even if a set looks like it has more objects matching the sets or counting may reveal a different result[2] (MP.1).

Instructional Strategies Continued: in the tens place should always be looked at first when determining which number is larger. They should be told that only when the digits in the tens place are equal they need to look at the ones place[3]. Have students participate in discussions involving questions like, “How did you get that?” and “Why is that true?” (MP.3, MP.6).

To make sure students understand what it means for a number to be greater or smaller. They should be referring to the number of tens, or the same number of tens with more/less ones. If students do not understand cardinality, they will struggle with comparing numbers in terms of digits (MP.5, MP.7, MP.8). Students should not have to use drawings to compare numbers at this level (MP.1, MP.2). Drawings should only be used when asked “How many more/less?” (MP.4).

Instructional Strategies Continued: done if the numbers were the same in the tens place (MP.7, MP.8).

Common Misconceptions: Students often have a hard time telling which group is bigger if they are not arranged in a orderly fashion or if the objects in the two groups being compared are a different size (e.g., pennies and quarters). Students may say the group with the bigger objects is larger because it looks larger (takes up more space). Direct students to the importance of counting the objects and not guessing.

Common Misconceptions: Many students do not understand the difference between “more” and “less.” Many students think that “less” is “more.” Extensive experience with a variety of contexts is needed to master the difference between “less” and “more.” Demonstrating the difference with manipulatives and numerous examples might have to take place before students can accurately compare numbers[2].

When using the symbols, $<$, $>$, and $=$ to compare numbers students are often taught to use an aid (pacman, alligator, bird, etc.).[1] While this is an effective aid in helping students draw the symbol in the correct direction, students often do not associate the real meaning and name with the symbol. It is important to stress the meaning of the symbols, (e.g., $<$ is less than) if using aids.

Common Misconceptions:

Order and Compare Numbers

Common Misconceptions Continued:	**Common Misconceptions Continued:**	**Common Misconceptions Continued:**
Assessment Procedure: Assessment 3: More/Less Trains	**Assessment Procedure:** **To be developed**	**Assessment Procedure:** **To be developed**
Vocabulary: Greater than Less than Equal (the same as)	**Vocabulary:** Greater than (>) Less than (<) Equal Compare Digit Greater Smaller	**Vocabulary:**

CCSS Domain	**Operations and Algebraic Thinking (K.OA)**	**Operations and Algebraic Thinking (1.OA)** *Students will demonstrate mastery of all previous skills and learn to:*	**Operations and Algebraic Thinking (2.OA)** *Students will demonstrate mastery of all previous skills and learn to:*
Addition and Subtraction	**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** 1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps) acting out situations, verbal explanations, expressions, or equations. 2. Solve addition and subtraction word problems, and add and subtract within 10, *e.g., by using objects or drawings to represent the problem.* 3. Decompose numbers less than or equal to 10 into pairs in more than one way, *e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).* 4. For any number from 1 to 9, find the number that makes 10 when added to the given number, *e.g., by using objects or drawings, and record the answer with a drawing or equation.* 5. Fluently add and subtract within 5.	**Add and subtract within 20.** 5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). 6. Add and subtract within 20, demonstrating fluency for addition and subtraction with 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12 , one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). **Represent and solve problems involving addition and subtraction.** 1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, *e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.* 2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, *e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.*	**Represent and solve problems involving addition and subtraction.** 1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, *e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.* **Add and subtract within 20.** 2. Fluently add and subtract within 20 using mental strategies (Mentioned in 1.OA). By end of second grade, know from memory all sums of two one-digit numbers.
	Instructional Strategies: It is important to note that students should not be exploring addition and subtraction in depth if they do not have an understanding of cardinality. If a student does not understand cardinality, the concept of addition and subtraction will likely be a process of memorizing and not a true understanding of the meaning of adding and subtracting. Provide students with real-world experiences of addition and subtraction. This is important so that students get exposed to how addition and subtraction relate to their lives. At this level students solve addition and subtraction problems through a variety of representations (MP.1, MP.4). They begin to use mathematical and non-mathematical language when asked to explain solutions[2] (MP.3, MP.5, MP.6). In the beginning, students may or may not use symbols in their representations, but it is important as teachers to use symbols in conjunction with modeling methods so that students get exposed to mathematical symbols relating to addition and subtraction[2].	**Instructional Strategies:** At this stage, students are no longer just solving problems with the result unknown (A + B = □, C - B = □) or both addends unknown (C = □ + □). They extend their knowledge of addition and subtraction to solving all twelve addition/subtraction subtypes found in the table on page 11 of this document (MP.7). Initially, the numbers within such problems are small enough so that students can still use drawings. Extending on the previous stage is the fact that students are now expected to write equations for all problems[2] (MP.1, MP.2, MP.4). Provide students with extensive and varied experiences that will help them develop a strong sense of numbers and number operations. Make sure that students are truly understanding addition and subtraction and not just following rules and procedures. It can be easy for a student to remember that 2 + 2 = 4, but they may be unaware of the conceptual meaning of 2 + 2 (MP.1).	**Instructional Strategies:** Students at this stage should begin solving two-step addition problems. They should be solving all problem types shown in the table on page 11 of this document by using drawings and equations with a symbol for the unknown number to represent the problem (MP.4, MP.6). The problems should involve sums and differences less than or equal to 100 using the numbers 0 to 100. Make sure students get in the habit of checking answers to see if they make sense as an answer for the situation and question being asked[1] (MP.1, MP.2, MP.3). To get students more familiar and comfortable with word problems, have them write word problems for their classmates. Provide some information for them to include such as the answer to the problem and the operation being performed. Remind them that they can only use the numbers 0 to 100 and the sum cannot be bigger than 100. For example, ask students to write a word problem that involves

Instructional Strategies Continued: Work with students to develop rapid visual and kinesthetic recognition of numbers to five on their fingers[2] (MP.5). At this level, and following levels, the use of fingers for keeping track of numbers is helpful to understanding addition and subtraction (MP.4). However, it is important that students do not use their fingers as a method of *direct modeling* in later levels[2].

Giving students experiences with the equals sign on the left and operation on the right is also important in helping them realize that problems can be written in a number of ways, and yet both sides still have the same value. This connects to concepts of commutativity and algebraic thinking that follow in later stages[2] (MP.1, MP.7).

When students are adding and subtracting using numbers less than or equal to ten it is important that problems are given to them in context that is relevant to them (e.g., if they are not familiar with the game jacks, don't create or use a problem about it until they first understand what jacks are). Not understanding the context of a problem makes solving the problem even harder (MP.7).

As students are given experiences to compose and decompose numbers of ten or less using manipulatives and drawings they will begin to connect addition and subtraction to counting principles they learned previously such as "adding one is just the next counting word" (K.CC.4c) (MP.4, MP.5, MP.7,)[2]. Additionally, as they are given these experiences they will begin to understand the role and meaning of adding and subtracting and therefore begin to gain computational fluency[1]. They will begin to see more patterns and start to internalize representations (MP.2, MP.8).

By the end of this stage, students should understand the meaning of addition and subtraction and be able to fluently (meaning without representations) add and subtract within 5.

Instructional Strategies Continued: As students are given experiences with working with numbers and adding/subtracting. Encourage students to share their discoveries (MP.8). They will discover that the whole is made up of parts, which is connected to decomposing and composing numbers (MP.7, MP.8).

Once students have mastered adding and subtracting with smaller numbers, they are to extend the range of numbers they deal with (up to 20). Because they are getting exposed to larger numbers, drawings are no longer an effective method of solving problems. They are to count on when solving addition and subtraction problems. Counting on is an effective method because it avoids having to draw lengthy pictures or using fingers to show totals of more than 10. Counting on is an effective thinking strategy that involves seeing the first addend as embedded in the total and subsequently using counting and cardinality to reach an answer (MP.2, MP.4, MP.5).

Provide numerous opportunities for students to practice counting on with addition and subtraction[2]. Avoid having students count down. This is not only more difficult but it takes from the reinforcement that counting on provides in showing that subtraction is an unknown-addend problem. Encourage students to use their fingers to keep track of the number of counts used when counting on. For example, in 13 - 9 = □, a student might say niiiiiine, ten, eleven, twelve, thirteen. So they should have four fingers up, meaning the answer is four. The elongating of the first counting word is natural and it shows differentiation between the addend and the first counting word. Encourage this action[2] (MP.5).

Make sure students are asked to explain, write, and/or reflect on their problem-solving strategies (MP.3, MP.6, MP.8).

Instructional Strategies Continued: adding three numbers with 17 as the answer. Have students share, compare, and discuss their problems and strategies[1] (MP.3, MP.6). Since most students are still learning proficiency with addition and subtraction, most work with two-step problems should initially be with single-digit addends[2].

Building on the knowledge students should have developed in the previous stages, students by the end of this stage should be fluent in adding and subtracting numbers within 20. This means that students should be fast and accurate in solving and no longer rely on manipulatives and representations. To help students become fluent in adding and subtracting within 20 provide numerous opportunities to practice adding and subtracting within 20. Students should not just be memorizing facts divorcing them from their meaning. It is important that they become fluent by coming up with their own strategies and patterns[2] (MP.7, MP.8).

In addition to becoming fluent in adding and subtracting within 20, students are to become fluent in understanding the relationship between addition and subtraction. They should be able to develop the seven related equations each addition or subtraction equation has. Understanding the related equations will help students in understanding and solving addition/subtraction problems with an addend unknown[2] (MP.1, MP.7, MP.8). For example, when given the problem 3 + 6 = 9, students should be able to find:

6 + 3 = 9	9 = 6 + 3
9 - 3 = 6	6 = 9 - 3
9 - 6 = 3	3 = 9 - 6
	9 = 3 + 6

Common Misconceptions: Students often mistake certain vocabulary in word problems for the wrong operation. Often students assume that *more* always means to add and the words *take away* or *left* always mean subtract. However, this is not true. For example, in this take from/start unknown problem: *Sam took away 5 stamps he did not want from his collection and gave them to Alexa. Now Sam has 10 stamps **left**. How many stamps did Sam have to begin with?* The equation would actually be addition. Therefore it is important that when students say *take away/left* to refer to subtraction that they are corrected to say *minus* or *subtract*[1].

It is important that students work with pictorial representations. If students move from using manipulatives to writing numeral expressions or equations too soon they likely will not understand addition/subtraction conceptually. They will likely use finger counting and rote memorization to solve problems rather than developing an understanding of patterns and grouping. They will likely not understand how to break up larger numbers and/or group to add on or subtract from[1].

Common Misconceptions: Many students misunderstand the meaning of the equal sign. They believe that the equal sign just means that an answer comes before or after it. Students need to understand that the equal sign means "is the same as." Additionally, many students get used to seeing problems with the equal sign to the right of the operation. This just feeds in even further to the misconception that an equal sign means the "answer is coming up." Therefore, it is important that students see problems written in multiple ways.

As mentioned in the previous stage for addition/subtraction, many students will continue to have a hard time with the language in word problems. They might continue to think that *left* always means subtract or *more* always means add. Continue to work on language within word problems. The more experiences with word problems, the better the understanding.

Make sure that students do not always think that the smaller number gets subtracted from the bigger number. This throws off thinking when it comes to regrouping. For example, in

Common Misconceptions: Some students think they have finished solving two-step addition/subtraction problems after they have only completed the first step. Either they did not understand the problem, or they are only focused on finding any answer to a problem. Provide students with ample opportunities to solve a variety of two-step problems and develop the habit of reviewing their solutions after they think they have finished[1].

Some students become proficient in particular facts (e.g., adding with 5) but when these facts are mixed with others , they may revert back to counting as a strategy. For example, a student may know 3 + 5 = 8 fluently. However, when given the problem 3 + 5 + 4, they resort back to counting on. They ignore the efficient strategies they have learned and developed. Reinforce to students that even though there is more than one addend, strategies they have developed can still be used[1].

Pay attention to misconceptions mentioned in previous stages. Students may still be misunderstanding the equals

Common Misconceptions Continued:	**Common Misconceptions Continued:** 14 - 8, students may look at the ones digits and think 4 gets subtracted from 8, and 0 get subtracted from 1, to get 14 as the answer. Students need to relate place value and grouping to their steps for subtraction.	**Common Misconceptions Continued:** sign as meaning “the answer is coming up.” Also, students still may misunderstand the language in word problems. For Example, students still may assume that *left* means subtraction all the time.
Assessment Procedure: Assessment 4: Number Arrangements (K.OA.3). Assessment 5: Combination Trains (K.OA.5) Assessment 6: Hiding Assessment (K.OA.5)	**Assessment Procedure:** Assessment 5: Combination Trains (1.OA.5, 1.OA.6) Assessment 6: Hiding Assessment (1.OA.5, 1.OA.6) Assessment 7: Ten Frames (1.OA.5, 1.OA.6)	**Assessment Procedure:** Assessment 5: Combination Trains (2.OA.2) Assessment 6: Hiding Assessment (2.OA.2) Assessment 7: Ten Frames (2.OA.2)
Vocabulary: Addition Subtraction More Less Solve	**Vocabulary:** Addition Subtraction Sum Difference Counting on Equal sign	**Vocabulary:** Addition facts Word problem Equation

CCSS Domain			**Operations and Algebraic Thinking (2OA)** *Students will demonstrate mastery of all previous skills and learn to:*
Multiplication and Division			**Work with groups of objects to gain foundations for multiplication.** 3. Determine whether a group of objects (up to 20) has an odd or even number of members, *e.g., by pairing objects or counting them by 2s;* write an equation to express an even number as a sum of two equal addends. 4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
	Instructional Strategies:	**Instructional Strategies:**	**Instructional Strategies:** To help students determine odd and even number of members and build a foundation for multiplication, have them use manipulatives and sort items into two equal groups. Students should come to realize that some sets have no items left over and some sets have only one left over (MP.1, MP.4). Explain to the students that even numbers will have no items left and odd will have one item left when separated into two piles. For an even number of objects in a collection, show the total as the sum of equal addends (MP.7, MP.8). This lays a foundation for multiplication because multiplication is repeated addition. For example, 12 items can be sorted into two groups of 6 with none left over. This means it is an even number, so it can be written as a sum of equal addends (6 + 6 = 12). Repeated addition is multiplication (6 x 2 = 12). So, while students were not told they are doing multiplication, they are indirectly learning background knowledge for multiplication[1]. Exposing children at this stage to arrays is key in building the foundation for multiplication. A rectangular array is an arrangement of objects in horizontal rows and vertical columns. All rows contain the same number of items and all columns contain the same number of items. Have students use objects to build all the arrays possible with no more than 25 objects (MP.4). Have students discuss their thinking. Ask students to explain how they got their array (MP.3, MP.4).Their arrays should have up to 5 rows and 5 columns. Then, have students draw the arrays on grid paper and write two different

Instructional Strategies Continued:	**Instructional Strategies Continued:**	**Instructional Strategies Continued:** repeated addition equations for each array. One showing the total as a sum by row and one showing the total as a sum by columns[1] (MP.1, MP.2, MP.5) For example: ●●●● ●●●● 12 = 4 + 4 + 4 ●●●● 12 = 3 + 3 + 3 + 3 Have the children investigate what happens with the array and the repeated addition equations if it is rotated 90°. They should come to the conclusion that the two arrays may look different but they are the same[1] (MP.1, MP.7, MP.8). Ask students to think of a way to represent a full ten frame as an array. For example, the ten-frame can be represented as an array with 5 rows and 2 columns (MP.4). Have students count by rows to 10 and write the equation 10 = 2 + 2 + 2 + 2 + 2. Once students understand a ten-frame as a repeated addition equation, have them use two ten-frames (20), to practice counting by 2s up to 20. Write an equation that shows 20 equal to the sum of ten 2s[1] (MP.1, MP.2, MP.7).
Common Misconceptions:	**Common Misconceptions:**	**Common Misconceptions:** Some students struggle with creating arrays for odd (non-prime) numbers. They will create an array and have a row or column with less items than the rest. Stress to children that all rows need to be the same and all columns need to be the same. So, students may need to rearrange and use trial and error to come up with a correct array for an odd number.
Assessment Procedure: **To be developed**	**Assessment Procedure:** **To be developed**	**Assessment Procedure:** **To be developed**
Vocabulary:	**Vocabulary:**	**Vocabulary:** Array Column Row Grid paper

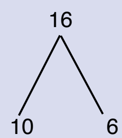
CCSS Domain	Number and Operations in Base Ten (K.NBT)	Number and Operations in Base Ten (1.NBT) *Students will demonstrate mastery of all previous skills and learn to:*	Number and Operations in Base Ten (2.NBT) *Students will demonstrate mastery of all previous skills and learn to:*
Place Value	**Work with numbers 11-19 to gain foundations for place value.** 1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, *e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8)*; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	**Understand place value.** 2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones - called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **Use place value understanding and properties of operations to add and subtract.** 4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. 5. Given a two-digit number, mentally find ten more or ten less than the number, without having to count; explain the reasoning used. 6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**Understand place value.** 1. Understand that the three digits of a three-digit number represent amounts of hundred, tens, and ones; *e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.* Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens - called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **Use place value understandings and properties of operations to add and subtract.** 5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. 6. Add up to four two-digit numbers using strategies based on place value and properties of operations. 7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three- digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. 8. Mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number 100-900.
	Instructional Strategies: Students should use objects, drawings, and equations to explore how "teen numbers" are made up of ten ones and some more ones[2]. It is important when first exploring the concept of tens that pre-grouped materials (such as a rod in base-ten blocks) be avoided. Students often have a hard time seeing that there are ten objects represented in that one pre-grouped material. Therefore they often attach words to the materials without truly knowing what they represent[1]. When decomposing numbers into ten ones and one, two, three... nine ones, hold discussions involving questions such as, "How did you get that?" and "Why is that true?" (MP.3, MP.6). Students should	**Instructional Strategies:** Composing and decomposing numbers into tens and ones helps in understanding number relationships and promoting mental computation[1]. Unlike in the previous level where students grouped ten individual items without seeing them as a unit. Students in this level learn that ten ones make a unit called ten. Students really begin to understand the breakdown of two-digit numbers and by the end of this stage are able to describe a number based on its tens and ones (MP.1, MP.3, MP.8). Drawings and place value cards help students see the connection between number words and/or numerals and their base-ten meanings (MP.1, MP.4).	**Instructional Strategies:** At this level, students extend their understanding of the base-ten system by viewing 10 tens as forming a new unit called a "hundred." The understanding of bundling in groups of 10 lays the groundwork for understanding the base-ten system. It helps students understand that the unit associated with each place is 10 of the unit associated with the place to the right[3]. The understanding that 100 is both 10 tens and 100 ones is critical to the understanding of place value. Using proportional models like base-ten blocks and bundles of tens along with place-value mats provides connections between physical and symbolic representations of a number. Using these models

Instructional Strategies Continued: first explore the concept of tens through groupable materials such as straws, paperclips, or a cup of beans (MP.4). A group of ten as a bundle makes more sense than a ten in pregrouped materials. By using groupable materials, students can create a group of ten items and then reuse it to represent other numbers (MP.5, MP.7, MP.8). By doing this, students can transition from the groupable to the pregrouped. Eventually, when students begin to understand the concept of a pregrouped set of ten, they can transition to representing the ten with a line (one tally mark) and the ones with dots[1] (MP.1, MP.2).

It is also important that the materials used to represent tens and ones are proportional. Meaning, the group of ten should physically be ten times larger than the model for one[1].

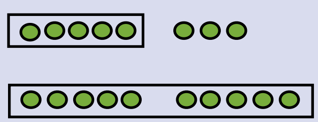
To help students further understand the break-down of teen numbers into ten ones and some more ones, layered place value cards, number-bond diagrams, and 5- and 10- frames (see diagram) can also help (MP.4). This will help students see that a number such as 18 is not “one, eight” but rather, “1 ten and eight ones.” The layered place value cards will help students see the 0 hiding under the ones place[2] (MP.2, MP.7, MP.3).

Number-bond Diagram and Equation



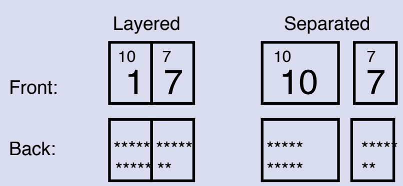
16 = 10 + 6

5- and 10-frames



Structuring the ten ones into patterns that can be seen as ten objects, such as two fives, will help students in developing quick recognition of numbers.

Place Value Cards



Instructional Strategies Continued: It is essential that students are given many opportunities to make tens using blocks, bundles of tens and ones, and ten-frames. While visually seeing the representation of a group of ten is important, making the connections among the representations, the numerals, and the words is key to future understanding of number properties and operation sense (MP.2, MP.4)[1].

Adding numbers within 100 provides a foundation for understanding that addition might require composing a 10. For example, in the problem 18 + 7, students might take 2 from the 7 and add it to the 18, to make a 20. This leaves the problem, 20 + 5 = 25. This is the prerequisite for higher level thinking in addition and subtraction. Allowing students to make drawings, use concrete objects, or place-value cards helps in making the connection between numerals and the base-ten meaning of the number. Additionally, it serves as a resource for explanation and discussion on methodology and place-value (MP.1, MP.6)[3].

Once students understand place value up to tens, they will begin to see patterns in numbers. Practice with addition and subtraction in terms of place value will help them to see patterns in adding and subtracting (MP.5). Encourage students to discuss patterns that they find (MP.3). When making drawings or using manipulatives, they will begin to see that adding ten more or taking from a number increases or decreases the tens place value by one (MP.1, MP.5, MP.6). They then will move into mentally computing ten more or ten less than a number without having to count by ones[3].

Students are not yet to go into computing the *differences* of two-digit numbers other than with multiples of ten. It is important not to go into computing the differences of two-digit numbers other than multiples of ten, because it allows for two-digit subtraction with and without decomposing to be taught in close succession in the next stage. Therefore helping children see the similarities between the two[3].

Instructional Strategies Continued: students can work on comparing numbers based on place value and identifying the value of their digits[1] (MP.2, MP.4, MP.5, MP.8)

Model three-digit numbers using base-ten blocks in multiple ways (MP.2, MP.4, MP.5, MP.7). For example, show how 342 can be 342 ones, 34 tens and 2 ones, or 3 hundreds, 4 tens, and 2 ones, or 30 tens and 42 ones. Use activities and games that have students match different representations of the same number[1]. Have students create their own representation of a number using manipulatives and discuss their process and reasoning (MP.3).

At this level, students should be fluent in adding and subtracting two, two-digit numbers. Additionally, they learn how to use strategies based on place value and properties of operations to solve addition and subtraction problems with up to four two-digit numbers or two three-digit numbers. When students add ones to ones, tens to tens, and hundreds to hundreds, they are using a method based on place value and the associative and commutative properties of addition[3] (MP.1)

Provide students with activities using connecting cubes, number grids, or other aids that will help them develop a strong understanding of number relationships, addition, and subtraction. Through these activities they can develop, share, and use efficient strategies for mental computation (MP.4). Students will gain computational fluency, using efficient and accurate methods for computing, as they come to understand the role and meaning of arithmetic operations in number systems. Efficient mental processes become automatic with use[1] (MP.1, MP.6, MP.8).

Composing and decomposing is more complex than the previous level because it requires another level of understanding place value. First, students must understand that a hundred is composed of 100 ones, and also 10 tens. Second, there is the possibility that both a ten and a hundred are composed or decomposed. For example, in computing 497 + 6 a new ten and a new hundred are composed. In computing 506 - 348, a ten and hundred are decomposed[3] (MP.1, MP.2, MP.6, MP.7).

Students will decompose and compose tens and hundreds when they develop their own strategies for solving problems where regrouping in necessary. They might use the make-ten strategy (46 + 8 = 50 + 4 = 54) or (53 - 7 = 50 - 4 = 46) because no ones are exchanged for a ten or a ten for ones (MP.1, MP.2, MP.6, MP.7).

Provide students with numerous examples that involve decomposing the tens or hundreds so that students can learn to identify right away when decomposing is needed.

Place Value	**Common Misconceptions:** Understanding the base-ten structure of teen numbers can be difficult for many children.Unlike many other numbers the number words for teen numbers do not clearly state their base ten structure. For example, the number eleven does not clearly state one ten and one one. The numbers in many other languages do state their teen numbers in the ten-one, ten-two, ten-three format. So, this may be especially difficult for English-language learners[2]. Additionally, many of the teen numbers state the ones digit first (e.g., nineteen) and makes children interpret that “teen” means “ten”[2]. Finally, prefixes such as “thir” and “fif” do not clearly say “three” and “five.” Using layered place value cards and manipulatives will help students understand teen numbers[2]. As mentioned above, students may also have a difficult time understanding that a group of ten things can be replaced by a single object. Using groupable materials will help students realize that a group of ten items can be replaced by a single object representing 10[1]. Make sure students are not just attaching names to materials. This misconception can interfere later on with the understanding of numbers beyond 19.	**Common Misconceptions:** When counting place value representations of numbers, students may misapply the procedure for counting on and treat the tens and ones as separate values. For example, in a representation of tens and ones such as ❙❙❙❙ ●●●● a student may count 10, 20, 30, 40, 1, 2, 3, 4, instead of 10, 20, 30, 40, 41, 42, 43, 44. To help students see that representations of tens and ones together represent a single value, provide them with frequent exposure to numbers and their representations. Practice counting in tens and ones patterns. Have them break down numbers into tens and ones and then count the tens and ones to see the connection between numbers and their place value representations.	**Common Misconceptions:** Some students misinterpret the values of digits in the tens and hundreds place. They often see the value as a single digit rather than a multiple of ten or hundred. For example, they may misinterpret the value of the 6 in 63 to be 6, not 60. Students need many experiences representing two- and three- digit numbers with groupable then pregrouped materials. When adding two-digit numbers, some students start with the digits in the ones place and record the entire sum. Then they continue to do so for the tens and the hundreds place. for example, 345 + 487 71212 Assess students’ understanding of a ten and provide more experiences modeling addition with grouped and pregrouped base-ten materials. Watch for students who think that the smaller digit always gets subtracted from the larger digit. Again, assess students’ understanding of a ten and provide more experiences modeling subtraction with grouped and pregrouped base-ten materials.
	Assessment Procedure: Assessment 7: Ten Frames (K.NBT.1)	**Assessment Procedure:** Assessment 7: Ten Frames (1.NBT.2a, 1NBT.2b, 1.NBT.2c) Assessment 8: Grouping Tens (1.NBT.2a, 1.NBT.2b, 1.NBT. 2c, 1.NBT.4) Assessment 9: Two-Digit Addition and Subtraction (1.NBT. 4, 1.NBT.5, 1.NBT.6)	**Assessment Procedure:** Assessment 12: Grouping 100s (2.NBT.1) Assessment 8: Grouping Tens (2.NBT.5, 2.NBT.6) Assessment 9: Two-Digit Addition and Subtraction (2.NBT.5, 2.NBT.6) Assessment 10: Hiding Assessment in Base Ten (2.NBT.5, 2.NBT.6) Assessment 11: One Hundred and Some More (2.NBT.7, 2.NBT.8) Assessment 12: Extension: Grouping 1000s (2.NBT.7, 2.NBT. 8)
	Vocabulary: Ones	**Vocabulary:** Ones Tens Mental math Place value	**Vocabulary:** Hundreds Regroup

CCSS Domain		**Operations and Algebraic Thinking (1.OA)** *Students will demonstrate mastery of all previous skills and learn to:*	**Numbers and Operations in Base Ten (2.NBT)** *Students will demonstrate mastery of all previous skills and learn to:*
Operation Comprehension		**Understand and apply properties of operations and the relationship between addition and subtraction.** 3. Apply properties of operations as strategies to add and subtract (no need for formal terminology). *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (associative property of addition.)* 4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.*	**Use place value understanding and properties of operations to add and subtract.** 9. Explain why addition and subtraction strategies work, using place value and the properties of operations (through drawings and objects).
	Instructional Strategies:	**Instructional Strategies:** Provide students with opportunities to investigate patterns in addition and subtraction. Provide them with manipulatives or specific numbers (e.g., 12, 3, 9) to discover how addends and the answer can be rearranged in a problem to come up with new problems. Allow for discussion on findings[1] (MP.1, MP.2, MP.3, MP.6, MP.8). Expand the student's work in addition to three or more addends to provide them with opportunities to change the order of addends and/or utilize groupings to make tens (MP.5). This will allow the connections between place-value models and the properties of operations for addition to be seen (MP.2). While students are not learning the formal terminology, understanding the commutative and associative properties builds flexibility for computation and estimation, a key element of number sense[1]. Put together/take apart problems with addend unknown give students the opportunity to see subtraction as the opposite of addition (MP.7, MP.8). Through strategies such as counting on, they may naturally discover how subtraction is an unknown-addend problem (MP.4, MP.5). This concept is one of the essential understandings for early algebra[2]. If students do not understand subtraction as an unknown-addend, it is	**Instructional Strategies:** It is important that students get practice on discovering their own strategies to solve problems so that they are not just memorizing procedures for solving problems (MP.1, MP.7, MP.8). These student-invented strategies should be shared, explored, recorded, and tried by others (MP.3, MP.4, MP.5, MP.6). Recording the expressions and equations in the strategies horizontally encourages students to think about the numbers and the quantities they represent instead of the digits (MP.2, MP.7). Not every student will invent strategies, but all the students can and will try strategies they have seen that make sense to them. Different students will prefer different strategies[1]. For example, to solve the problem 23 + 17 students may solve it like: 7 + 3 = 10 3 + 17 = 20 20 + 10 = 30 or 20 + 20 = 40 30 +10 = 40 Number talks are key in helping students explain and think about how addition and subtraction strategies work. In a number talk, children are inspired to think and make of mathematics on their own. Number talks are different from a

Operation Comprehension

Instructional Strategies Continued:	**Instructional Strategies Continued:** important to continue to work on this concept through it is a variety of methods. Games, modeling, and real-world connections may help students understand the connection between addition and subtraction[1] (MP.1, MP.4).	**Instructional Strategies Continued:** lesson in that during a lesson, the teacher usually has a particular goal set up for her class. And in a lesson, the teacher creates particular relationships for the students that are as obvious as possible. In a Number Talk, a teacher presents children with a problem and children are asked to solve the problem through means that make sense to them. They are never trying to figure things out the way the teacher wants, rather they are totally engaged in their own thinking. In Number Talks it is important that children are asked to explain their thinking and reason as to how they got the answer they did.
Common Misconceptions:	**Common Misconceptions:** Students often think that the commutative property applies to subtraction. To fix this misconception, have students practice the commutative property using manipulatives and/or drawings. Ask them to try the property with addition and then try it with subtraction. Ask them to explain their findings. Guide them to conclude that the commutative property does not work for subtraction[1]. First graders might have informally encountered negative numbers in their lives, so they think they can take away more than the number of items in a given set, resulting in a negative number below zero. Provide many problems situations where students take away all objects from a set, e.g. 19 - 19 = 0 and focus on the meaning of 0 objects and 0 as a number. Ask students to discuss whether they can take away more objects than what they have[1].	**Common Misconceptions:**
Assessment Procedure: **To be developed**	**Assessment Procedure:** Assessment 5: Combination Trains (1.OA.3) Assessment 6: Hiding Assessment (1.OA.4)	**Assessment Procedure:** Assessment 11: One Hundred and Some More (2.OA.9) Assessment 12: Extension: Grouping 1000s (2.OA.9)
Vocabulary:	**Vocabulary:** addend	**Vocabulary:**

CCSS Domain		**Operations and Algebraic Thinking (1.OA)** *Students will demonstrate mastery of all previous skills and learn to:*	
Algebraic Thinking		**Work with addition and subtraction equations.** 7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.* 8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ – 3, 6 + 6 = □.*	
	Instructional Strategies:	**Instructional Strategies:** Students need to see an equal sign as not just a symbol that means an answer is following. An equal sign means that the value to its right will be the *same* as the value to its left. Provide students with opportunities to explore equations and their values. Have them use manipulatives and number balances to model equations and compare the values on either side of the equals (MP.1, MP.4, MP.5). Include equations that show the identity property (8 = 8), the commutative property of addition (7 + 3 = 3 + 7), and the associative property of addition (4 + 8 + 6 = 10 + 8). Proper terminology should not be used[1] (MP.7). Allow for discussion of findings and the meaning of the equal sign (MP.1, MP.3, MP.6, MP.8). Be sure to also present equations in non-traditional ways. Like, 18 = 20 - 2 and 6 + 7 = 17 - 4 (MP.7). Have students determine whether the statements are true or false and explain why they are either true or false[1] (MP.1, MP.2, MP.3, MP.5, MP.6). When having students solve for unknown values have students use drawings, words, and numbers (MP.1, MP.4, MP.5). Make sure to stress the relationship between addition and subtraction. Provide examples of rearranging numbers to create an easier to solve problem. Graphic organizers such as number triangles can also be used as an aide[1] (MP.2, MP.4, MP.5).	**Instructional Strategies:**

Instructional Strategies Continued:	**Instructional Strategies Continued:**	**Instructional Strategies Continued:**
Common Misconceptions:	**Common Misconceptions:** Many students think the equal sign is just a symbol that separates an operation on the left and its result on the right. They do not see the equals sign as a symbol that means "the same as." Essentially, they see the equals sign as an arrow pointing in the direction of what the operation becomes. Stress the meaning of an equal sign in classroom activities and lessons. Students also have a hard time understanding problems written in non-traditional ways. For example, a problem written like 11 = □ - 4 might be solved 11 - 4 = □. Providing examples of problems with the solution on the left will help students realize that the equals sign means that the left should be the same as the right[1]. Showing problems written in multiple ways will help students gather meaning of the equals sign as well as the relationship between numbers within a single problem.	**Common Misconceptions:**
Assessment Procedure: **To be developed**	**Assessment Procedure:** **To be developed**	**Assessment Procedure:** **To be developed**
Vocabulary:	**Vocabulary:** True False Unknown	**Vocabulary:**

CCSS Domain	Measurement and Data (K.MD)	Measurement and Data (1.MD) *Students will demonstrate mastery of all previous skills and learn to:*	Measurement and Data (2.MD) *Students will demonstrate mastery of all previous skills and learn to:*
Measurement	**Describe and compare measurable attributes.** 1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. 2. Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	**Measure lengths indirectly and by iterating length units.** 1. Order three objects by length; compare the lengths of two objects indirectly by using a third object. 2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	**Measure and estimate lengths in standard units.** 1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. 2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. 3. Estimate lengths using units of inches, feet, centimeters, and meters. 4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **Relate addition and subtraction to length.** 5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, *e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.* 6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
	Instructional Strategies: At this level students are gaining a foundation for their study of statistics and probability. Laying the foundation down for spatial measurement is key in understanding direct measurement in future stages. For example, problems such as “is the bookcase taller than the door? How do you know?” lays the foundation for the direct measurement problems like “How much taller is the bookcase than the door?” (MP.2, MP.3, MP.7). At this level the *measurable* attributes of an object is initially given, but eventually students will be determining them on their own. Students should be given opportunities to compare objects or shapes directly. They then can explore the objects height, weight, volume, etc., through hands-on methods (MP.4). For example, if asked when comparing the volume of two containers: Which container will hold the most? The least? (MP.1, MP.2, MP.3) Students can then explore the volumes	**Instructional Strategies:** Building on the previous stage, students are no longer just directly comparing measurable attributes. They are now measuring measurable attributes with non-standard units. In order to correctly measure an object, a student needs to understand what a unit of measure is and how it can be used to measure an object. At this level, it is important that instruction starts with informal units of measurement (paperclips, beans, straws, sand etc). This will allow for the students to focus on the attributes being measured and not on what units are being used. Working with informal or nonstandard units is a prerequisite for measuring with standard units[1]. Make sure to provide examples where many copies of the unit is needed to find the length of an object. For example, to measure the length of a room with the unit of a footprint, students will need to line up many copies of the footprint with no gaps to find the length of the room. This will help students	**Instructional Strategies:** At this level students are transitioning from using nonstandard units to measuring with the standard units of inches, feet, centimeters, and meters. The measure of length is a count of how many units are needed to match the length of the object or distance being measured[1]. Make sure to provide students with experiences measuring lengths with appropriate tools so that they become familiar with using standard units (MP.5). Once they have been given experiences using standard units they will be able to estimate lengths (MP.5, MP.8). Use language that reflects the approximate nature of measurement such as the length of the room is about 27 feet[1] (MP.1, MP.4) Have students practice measuring the same object with different-sized units. Discuss what they noticed. Children should realize that the smaller units yield a higher measurement and vice versa[1] (MP.1, MP.7).

Measurement

Instructional Strategies Continued: using a variety of different methods. They might describe the capacity based on the visual size of the sides of the containers (MP.5, MP.6, MP.8). The objective is to have students realize that all objects have measurable attributes[1] (MP.1, MP.4, MP.7).

It is important to note that at this stage students are not directly measuring anything. Not even with non-standard units (e.g., paperclips). Students are to just make comparisons about the measurable attributes of objects (e.g., Look at the lengths of these two students, how do they compare?)

Instructional Strategies Continued: later on realize that sometimes the length of one ruler is not enough to measure an object (MP.1). So, they will have to either line up multiple rulers or use the same ruler multiple times[1] (MP.2, MP.4, MP.5, MP.6, MP.7, MP.8).

Make sure students realize that the spaces of a unit of measurement represent the length and not the marks or numbers on a ruler. To have students practice this, let them use small objects such as paperclips or pennies to measure the length of an object. Have them make marks on the endpoints of the object and then color in the spaces. They can then count the number of spaces to find the length of the object (MP.2, MP.4, MP.5, MP.6). Provide students with opportunities to discuss and evaluate findings (MP.1, MP.3).

Once students seemed to have mastered using non-traditional units, have them practice using rulers. At this stage there is no need to pay attention to units. Be sure to encourage students not to use the end of a ruler as the smallest point. Often there is a gap on a ruler before the zero mark and students just line up the beginning of an object with the end of the ruler and they do not compensate for the gap. Either point out the gap on the ruler and the importance of starting right on zero, or teach the students how you can measure an object from a different starting point and compensate for it through subtraction. Make sure to practice measuring objects longer than a ruler[1] (MP.1, MP.5, MP.6).

Help students to discover how they can use indirect reasoning to compare measurements indirectly. For example, given three objects, A, B, and C, have students compare the lengths of A and B then B and C. Based on their findings, have students use reasoning to determine how the length of all three objects compare[1] (MP.1, MP.6, MP.7, MP.8).

Instructional Strategies Continued: Stress to students to always make an estimate of a length before measuring (MP.5, MP.8). Having a student estimate before measuring will make them focus more on the process of measuring, the attribute being measured, and the length units. After finding a measurement, have students discuss their estimates, their procedure for finding the correct measurement, and the difference between their estimates and the measurements[1] (MP.2, MP.3, MP.6).

Connect number lines to whole-number units on rulers, yardsticks, meter sticks and measuring tapes. Stress the similarities between the two and how they both can be used to find sums and differences between numbers. Demonstrate how number lines are created and how they can be used to solve addition and subtraction problems. Show addition and subtraction on a number line using curved line segments between the numbers. Drawing curves or "hops" between numbers will help students focus on a space as the length of a unit and the sum or difference as a length (MP.4, MP.6).

Have students practice using measuring tools to model different representations for whole-number sums and differences less than or equal to 100 using the numbers 0 to 100[1] (MP.4, MP.5).

Provide one- and two-step word problems that include different length measurements made with the same unit (inches, feet, centimeters, and meters). Students add and subtract within 100 to solve problems for these situations: adding to, taking from, putting together, taking apart, comparing, and with unknowns in all positions. Students use drawings and write equations with a symbol for the unknown to solve the problems (MP.1, MP.2, MP.4, MP.5, MP.6). Have students explain their thinking (MP.3, MP.6).

Measurement can also be used to integrate and reinforce skip counting. A meter stick can be used to view units of ten (10cm) and hundred (100cm), and is a good tool to practice skip counting by 5s and 10s (MP.5, MP.7, MP.8).

Common Misconceptions: Some students think that all measurement descriptors are permanent and not relative. For example, Sally knows she is "bigger" than her little brother, so she thinks she is "bigger" than everyone. She might think she is even bigger than her parents or teacher. Therefore, it is important to talk about relative sizes of objects and how an object can be both "bigger" and "smaller" depending on what they are compared to.

Common Misconceptions: Sometimes students mistake measuring for a counting task. When using a ruler, they may count the markings on a ruler rather than the spacings. It is important that students see the spaces as the unit of measure and not the lines as the unit of measure. Experience with individual units of measure or with rulers with numbers in the center of spaces may help[1].

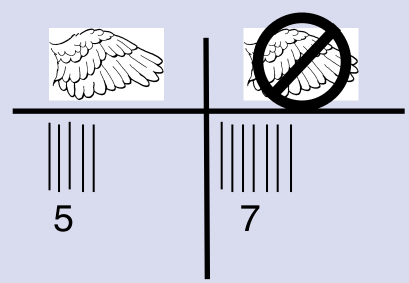
Another common misconception is that measuring begins at the beginning of a ruler. It is important to point out to students that zero is often not directly on the end of a ruler.

Common Misconceptions:

Measurement			
	Common Misconceptions:	**Common Misconceptions:**	**Common Misconceptions:**
	Assessment Procedure: **To be developed**	**Assessment Procedure:** **To be developed**	**Assessment Procedure:** **To be developed**
	Vocabulary: Length Weight Taller Shorter Longer Larger Smaller	**Vocabulary:** Unit Ruler	**Vocabulary:** Ruler Meterstick Measuring tape Foot Meter Inches Centimeters Number line

CCSS Domain		**Measurement and Data (1.MD)** *Students will demonstrate mastery of all previous skills and learn to:*	**Measurement and Data (2.MD)** *Students will demonstrate mastery of all previous skills and learn to:*
Time and Money		**Tell and write time.** 3. Tell and write time in hours and half-hours using analog and digital clocks.	**Work with time and money.** 7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. 8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*
	Instructional Strategies:	**Instructional Strategies:** Telling time can be a hard concept to grasp, but it is important to note that students are not expected to know all of the minute hand positioning at this stage. Since counting by 5s is not an expectation at this stage, focus is on recognizing where the minute hand is at the hour and half-hour[1]. What is important for students to understand is that the little hand represents the hour and the big hand represents minutes before and after an hour. The big hand focuses on distance it has gone or needs to go, while the little hand is a direct measurement[1] (MP.5). To help students understand telling time, initially use a one-handed clock with just a big hand. Have students discuss what is happening to the little hand when the big hand is at the hour or half-hour (MP.2, MP.4, MP.5, MP.7). For example, when the big hand is on the half-hour, the little hand is between two numbers but the time is still in the earlier number. After students understand the clock hand placements for an hour and a half-hour, have students practice with a clock that only has the little hand (MP.2, MP.4, MP.5, MP.7). Students can predict the placement of the big hand based on the little hand. Have students check answers with a two-handed clock (MP.1, MP.6, MP.8). Allow opportunities for students to discuss time and explain their reasonings when telling time (MP.3). Have students tell time frequently throughout the day. Provide lots of exposure to clocks and telling time.	**Instructional Strategies:** Students expand on the knowledge they learned about telling time in the previous level to now telling time to the nearest five minutes using a.m. and p.m[1]. Before jumping into telling time to the nearest five minutes, students first need to have a good understanding of skip counting by 5s. Students that do not know how to count by 5s need to continue to work on skip counting. Analog clocks can be used as an aid to skip count by 5s by having students use the numbers on a clock as tracking points for each skip count (MP.4, MP.5). However, it is important to make sure that counting by 5s does not just become rote memorization or a memorization based on association with the numbers on a clock. A student needs to understand what it means to count by 5s and see it not just as a counting pattern to memorize. Once students have a good understanding of counting by 5s, they can apply what they learned about the hands on a clock in the previous level to tell time to the nearest five minutes (MP.7). Frequent practice and references to time will help in the development of this concept. Instead of telling students what time it is, ask students to tell you what time it is during the day. Connect it with the daily class schedule (MP.1). Frequently make reference to a.m. and p.m. throughout the day as well. Have children explain how they figured out what time it is (MP.3, MP.6). Even though the concept of money does not come up in the standards until this level, it builds off of standards from this level and previous levels. Help students learn money concepts and solidify their understanding of other topics by providing activities where students make connections between them (MP.2). For example, the value of a dollar bill as 100 cents can be linked to the concept of counting by 100s to 1000. Use play money - nickels, dimes, and dollar bills to skip count by 5s, 10s, and 100s (MP.4, MP.5). Reinforce place value concepts with the values of dollar bills, dimes, and pennies[1] (MP.7, MP.8). Encourage students to total up coin values starting first with dollars and quarters, then dimes, nickels, and pennies.

Time and Money	**Instructional Strategies Continued:**	**Instructional Strategies Continued:**	**Instructional Strategies Continued:** Incorporate the context of money to find sums and differences less than or equal to 100 using the numbers 0 to 100. Provide students with addition and subtraction one- and two-step word problems involving money situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. Students use drawings and equations with a symbol for the unknown number to represent the problem (MP.2, MP.5, MP.7). Encourage students to practice their mathematical communication skills and have them explain their thinking and reasoning for arrived answers (MP.3, MP.6). The dollar sign, $, is used for labeling whole-dollar amounts without decimals, such as $34[1] (MP.1). Provide frequent practice on the relationship between the values of a penny, nickel, dime, quarter, and dollar bill. Activities where children get to exchange coins or a dollar bill for equivalent amounts of other coins will help in understanding the relationship between coins and dollar bills (MP.2, MP.8).
	Common Misconceptions:	**Common Misconceptions:** Since digital clocks do not allow for students to see the relationship between times, digital clocks should only be used as an additional form of support. Students can predict what the display on a digital clock will look like based on an analog clock's display (only to the hour and half-hour). Students often confuse the hour hand and the minute hand. Many students think that because we say the hour first, the bigger hand represents the hour. To help students remember that the hour hand is the small hand and the minute hand is the big hand, provide them with frequent exposure.	**Common Misconceptions:** Some students confuse the hour and minute hands. So for a time of 5:35 they may say 7:25. Also, some students will same the numeral that a hand is closest to instead of the minute correspondence (e.g., 8 instead of 40). For example, if the time was 5:35 a student might say 5:07 or 7:05. Pay attention to students' understanding of the roles of the minute and hour hands and the relationship between them. If students often confuse the hour and minute hand, provide remediation and just focus on the movement and features of the hands[1]. Some students overgeneralize the value of coins when they count them. They might count them as individual objects. Also some students think that the value of a coin is directly related to its size, so the bigger the coin, the more it is worth. Provide representations of coins and their values in manipulatives such as five-frames and ten-frames. For example, place a nickel on top of five-frames that are filed with pictures of pennies. Likewise, attach pictures of dimes and pennies to ten-frames and quarters to 5 x 5 grids filled with pennies. Have students use these materials to get familiar with coin values[1].
	Assessment Procedure: **To be developed**	**Assessment Procedure:** **To be developed**	**Assessment Procedure:** **To be developed**
	Vocabulary:	**Vocabulary:** Clock Hour Half-hour Minutes Digital Analog	**Vocabulary:** A.M. P.M. Dollar (and dollar symbol/$) Quarter Cents Penny Nickel Dime

CCSS Domain	Measurement and Data (K.MD)	Measurement and Data (1.MD) *Students will demonstrate mastery of all previous skills and learn to:*	Measurement and Data (2.MD) *Students will demonstrate mastery of all previous skills and learn to:*
Represent and Interpret Data	**Classify objects and count the number of objects in each category.** ☆ 3. Classify objects into given categories; count the number of objects in each category and sort the categories by count (limit counts to less than or equal to ten).	**Represent and interpret data.** ☆ 4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**Represent and interpret data.** ☆ 9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. 10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put- together, take-apart, and compare problems using information presented in a bar graph.
	Instructional Strategies: It is important that students realize all the different ways objects can be classified. There is not just one way to classify an object. Have them explore objects around the classroom and brainstorm all the different ways they can classify objects within the classroom[1]. Discuss with students their findings and reasonings (MP.1, MP.2, MP.3, MP.5, MP.6, MP.7). Also connect classification to other subject areas. Classification can be seen in all subject areas. For example, in reading, words can be classified based on vowel sounds, syllables, beginning letter, etc. The more students practice classification, the better they will get at determining their own measurable attributes to classify something (MP.4). After classifying and sorting objects students should count the number of objects in each category (MP.4). Then, if they have a grasp of cardinality and comparing numbers, have them talk about their finding referring to the numbers (MP.2, MP.3, MP. 6, MP.8). Have students also practice with classification by guessing the rule for already sorted items (MP.1).	**Instructional Strategies:** In the previous stage, students learned how to recognize the number of objects in a group and then compared it to the number of objects in another group. Unlike the previous stage, where groups were either made for the student or specific criteria was given for sorting, students now begin to sort items by their own criteria. Students should be asked to sort a collection of items by up to three categories. Certain objects may still warrant the need for sorting categories to be given[4]. Discuss and analyze findings of sorts (MP.1, MP.3, MP.7). Reinforce previous concepts, by asking students about the number of items in each category as well as which category has more items (MP.2). Also, have students practice addition by adding up the total number of items (MP.1). The total number of items should be less than or equal to 20, since addition and subtraction up to 20 is the focus of this level[1]. This concept should also be connected to geometry by having students sort collections of geometric shapes. Students should then be questioned on their sorts[1]. After students have had experience sorting objects, they may begin to create graphic representations of the sorts. They  	**Instructional Strategies:** At this stage students expand on their graphing knowledge to move from making tally counts of categories or cluster graphs to making bar graphs and picture graphs. A bar graph representing categorical data displays no additional information beyond the category counts. The bars are just a way to make the category counts easier to visually interpret. The hardest part for children is not collecting the data, but properly creating and labeling bar graphs[4]. At first students should create picture graphs where each row or bar consists of countable parts (MP.4). These graphs show items in a category and do not have a numerical scale. For example, if students were counting the different color eyes in his/her class, the graph would show eyes lined up end to end horizontally or vertically. Students would then just count how many eyes were in a row or column[1]. Then practice using scaled picture graphs where one picture may represent 2,3,4... counts which is represented in a key (MP.2, MP.5). Make sure students label the categories and the title of the graph (MP.1, MP.6). After students learn to make picture graphs, move to making bar graphs with a numerical scale (MP.4). Demonstrate how a bar graph is just like a picture graph but it has a number scale. Refer to the number scale as a count scale and discuss how it is similar to a number line diagram[4]. Practice labeling the scale on bar graphs using graph paper (MP.1, MP.6). Line plots are useful tools for collecting data because they show the number of things along a numeric scale. They are made by simply drawing a number line then placing an X above the corresponding value on the line that represents each piece of data. Line plots are essentially bar graphs with a potential bar for each value on the number line[1] (MP.1)

Represent and Interpret Data

Instructional Strategies Continued:

Instructional Strategies Continued: may create tables with marks for the number of items in each category. Or, they may create picture graphs in which one picture represents one object (though picture graphs are not an expectation until the next level). It is important that all of these forms of representation get modeled by the teacher several times before students create their own. Have students discuss and analyze representations of data (MP.1, MP.2, MP.3, MP.4, MP.5, MP.6, MP.7, MP.8).

Be sure to provide numerous opportunities for students to create questions, collect data, organize data, and interpret results.

Instructional Strategies Continued:
Line Plot:

					X	X				X		
	X		X		X	X		X		X		
	X	X	X		X	X		X		X		X
10	11	12	13	14	15	16	17	18	19	20	21	22

At this level students are generating a set of data on their own. For example, students might measure the length of their hand and record values to make a class data chart (MP.4, MP.5). Students need to understand that measurement data gets represented in a line plot (MP.2). Instruct students on how to make a line plot by first making a number line that ranges from the greatest and least values and then making marks above the line for each data piece[4] (MP.4, MP.6).

Discuss the similarities and differences between bar graphs, picture graphs, and line plots. Discuss how line plots allow one to see gaps between data points (e.g., four data points on 67 and one on 69, but none on 68), while in bar graphs there are no "gaps" between categories (e.g., green and blue)[4] (MP.1, MP.3, MP.6, MP.7).

Pose questions that use information in the graphs. Ask questions that involve simple put together, take-apart, and compare problems like those found in the table on page 12[1].

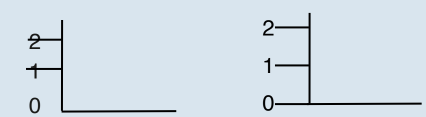
Common Misconceptions: Students may have a hard time narrowing down the method of classification in presorted objects. They may give a set of criteria that is too broad. For example, if some shapes were sorted into piles of triangles and circles, a student might say the classification of the triangle pile is "shapes" rather than triangles. Help students see that sometimes they need to look closer at the objects in a set to see if they have more specific details in common.

Common Misconceptions: Students may have a hard time coming up with the criteria to sort a group of object by. They may choose criteria too broad or too narrow. Or, they may choose criteria such that an object fits into two criteria making the sort difficult. Students that struggle with coming up with criteria to sort objects need to still be directed to how groups should be sorted. Some students may still need the criteria for sorting told directly to them, while other might just need a direction (e.g., look at the colors of the items).

Common Misconceptions: It might be natural for a student to want to represent measurement data with a bar graph, but measurement data points (e.g., 25in.) are not categories and therefore cannot be put in a bar graph. They should be represented through a line plot.

Students often have trouble labeling a scale correctly on graph paper. They often write the scale numbers within the spaces on the grid rather than next to the tics where the horizontal and vertical lines meet. Stress this when instructing students on how to make bar graphs.

e.g.



Incorrect **Correct**

Assessment Procedure:

To be developed

Assessment Procedure:

To be developed

Assessment Procedure:

To be developed

Vocabulary:
Sort

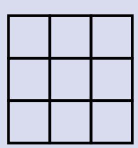
Vocabulary:
Number count
Category
Data
Collect

Vocabulary:
Picture graph
Bar graph
Line plot
Scale

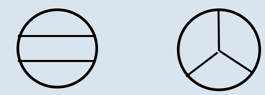
CCSS Domain	Geometry (K.G)	Geometry (1.G) *Students will demonstrate mastery of all previous skills and learn to:*	Geometry (2.G) *Students will demonstrate mastery of all previous skills and learn to:*
Shape Recognition	**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** 1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. 2. Correctly name shapes regardless of their orientations or overall size. 3. Identify shapes as two-dimensional (lying in a plane, "flat") or three- dimensional ("solid"). **Analyze, compare, create, and compose shapes.** 4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	**Reason with shapes and their attributes.** 1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	**Reason with shapes and their attributes.** 1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces (through direct comparison). Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
	Instructional Strategies: Help students develop spatial sense by connecting geometric shapes to their everyday lives[1]. Have them locate two-dimensional and three-dimensional shapes in their classroom and everyday lives (MP.7, MP.8). In addition to having students locate and discuss shapes in their everyday lives also have them describe the relative positions of shapes to objects around it (e.g., "The circle (clock) is over the rectangle (white board))[1] (MP.1, MP.3). Make sure that students recognize that two-dimensional means "flat" and three-dimensional means "solid." Using manipulatives, help students realize that two-dimensional shapes are found in three-dimensional shapes (MP.2, MP.5). When talking about shapes, be sure to frequently refer to the number of corners and sides on a shape. Make connections/ comparisons to the number of corners and sides in other shapes[1]. Have Students model and discuss their knowledge of sides, faces, and corners using manipulatives (MP.4, MP.6, MP.7, MP.8). Use sides, faces, and vertices (corners) of shapes to practice counting and reinforce the concept of one-to-one correspondence (MP.1, MP.2, MP.5).	**Instructional Strategies:** Students should practice describing a shape based on defining attributes. Allowing them to use manipulatives and geoboards can help children understand defining and non-defining attributes (MP.4, MP.5). Asking children to create or point out shapes based on defining attributes will help them realize their importance (MP. 1, MP.2, MP.7, MP.8). They will realize how much more specific defining attributes are in identifying shapes[1]. Students can also describe shapes while the teacher records the defining attributes mentioned. Additionally, pattern blocks can serve as a method for students to model defining attributes. They can sort them based on defining attributes. Make sure students are not sorting shapes based on non-defining attributes (e.g., color, size)[1] (MP.4, MP.5, MP.7, MP. 8). Discuss how different shapes can have similar defining attributes. Have students determine whether given attributes are defining or non-defining (MP.3, MP.4).	**Instructional Strategies:** Similar to the previous stage, at this stage students continue to work on describing shapes based on their attributes. Differences are seen in the fact that now students are using defining attributes to describe and name specific shapes. In the previous stage students learned the difference between defining attributes and non-defining attributes, but were not expected to name shapes based on attributes. By the end of this stage, students can both recognize, describe, and draw shapes based on defining attributes (MP.1, MP.3, MP.6, MP.7, MP.8). Question students on their reasoning. While students were already expected to identify numerous shapes in previous stages, in this stage students are no longer just identifying, but also drawing and analyzing. Use shape attributes identifying shapes and objects to help students get comfortable with referring to shape attributes. For example, if asking for students to get something out of a box, use it as an opportunity to ask students what shape is a box (a cube) and what are some of its defining attributes (e.g., 6 equal faces) (MP.1, MP.2, MP.4, MP.5, MP.7). Frequent exposure and references to shapes such as triangles, quadrilaterals, pentagons, hexagons, and cubes will help students recognize how defining attributes make unique shapes (MP.4, MP.7, MP.8).

Shape Recognition

Instructional Strategies Continued:	**Instructional Strategies Continued:**	**Instructional Strategies Continued:** Provide students with experiences that require drawing regular shapes. Students are not expected to draw shapes perfectly, but should show evidence of the defining attributes of shape (e.g., three corners and three sides) (MP.4, MP.5, MP.6).
Common Misconceptions: Students often have a hard time distinguishing between two-dimensional names and three-dimensional names for shapes. They may call a “cube” a “square” or a “sphere” a “circle.”[1] Work with students often so that they can understand that a two-dimensional shape is part of a three-dimensional shape, but they have different names. Additionally, many students have a hard time recognizing shapes when they are in different orientations. A student may recognize a square when it’s bottom edge is “straight” (■) but may not recognize it when it is tilted so that a corner is facing downward (◆) . To help students recognize shapes in manipulated formats, frequently talk about the properties of shapes and provide them with the opportunity to flip and rotate different shapes. After flipping and rotating shapes, talk about how the shape and the properties of the shape are still the same.	**Common Misconceptions:** Like in the previous level, some students may have a hard time recognizing shapes when they are in different orientations. Be sure to continue to let students experience shapes in different orientations.	**Common Misconceptions:** Students may still think a shape is changed by its orientation as mentioned in the previous levels. This is why it is so important to have young students handle shapes and physically feel that a shape does not change regardless of the orientation[1].
Assessment Procedure: **To be developed**	**Assessment Procedure:** **To be developed**	**Assessment Procedure:** **To be developed**
Vocabulary: Above Below Beside In front of Behind Next to Flat shape/two-dimensional Solid shape/three-dimensional Square Circle Rectangle Triangle Hexagon Cube Cone Cylinder Sphere Corner/vertices Side	**Vocabulary:** Defining attribute Corner Side Trapezoid Cube Right rectangular prism Cone Cylinder	**Vocabulary:** Angles Faces Sides Quadrilaterals Pentagon Hexagon

CCSS Domain	Geometry (K.G)	Geometry (1.G) *Students will demonstrate mastery of all previous skills and learn to:*	Geometry (2.G) *Students will demonstrate mastery of all previous skills and learn to:*
Shape Manipulation and Spatial Reasoning	**Analyze, compare, create, and compose shapes.** ☆ 5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. 6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	**Reason with shapes and their attributes.** ★ 2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape (students do not need to know formal names of shapes). 3. Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	**Reason with shapes and their attributes.** ☆ 2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. 3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
	Instructional Strategies: Have students practice drawing and making shapes by providing them with materials such as clay, construction paper, popsicle sticks, etc (MP.5). Students are not expected to perfectly draw all shapes, but all the attributes of the shape need to be present (e.g., a triangle needs to have three sides and three angles) (MP.4). Also provide students with a variety of shape manipulatives and real-world objects so that they can create new and larger shapes. For example, using two triangles to make a rectangle (MP.1, MP.2, MP.5). Expose them to the fact that they can use multiple smaller shapes to recreate a larger version of the shape (e.g., they can use nine one-inch squares to make a three-inch square). Make sure to ask students to describe the new shape formed in terms of sides and corners (MP.3, MP.6, MP.7, MP.8).  Giving students the opportunity to play with shapes and put different shapes together also provides them with opportunities to see that familiar objects are made out of a variety of shapes (e.g., a simple house diagram can be broken up into a square and a triangle) (MP.4, MP.7, MP.8).	**Instructional Strategies:** Students should use a variety of manipulatives and real-world objects to create larger shapes out of smaller ones. By using manipulatives such as paper shapes, color tiles, pattern blocks, small boxes, etc, students can explore how shapes can be combined to make new shapes (MP.4, MP.5). This also provides them with exposure to the variety of different two-dimensional and three-dimensional shapes there are and how shapes can be similar or different. Have a discussion on findings (MP.3, MP.7, MP.8). To provide students with experience partitioning shapes into equal shares, allow students to fold shapes made from paper to physically feel the shape and form the equal shares (MP.5). When having students fold the shapes refer to the way to which the paper is being folded (halves or fourths). After folding the paper into halves they should look at how their paper has been partitioned. Repeat after folding into fourths. Discuss the change in the size of the parts from halves to fourths (MP.1, MP.2, MP.3, MP.4, MP.6, MP.7, MP.8). After students have been given experience with partitioning using manipulatives, they should work on drawing lines in shapes to partition them into halves and fourths (MP.1, MP. 2, MP.4, MP.7, MP.8).	**Instructional Strategies:** Partitioning rectangles into rows and columns of the same size is a prerequisite to multiplication and finding the area of rectangles. Give students grid paper and ask them to draw a rectangle of a specific size (e.g., 2 units by 2 units). Then, have students guess and check how many 1 unit by 1 unit squares will be inside the rectangle. Next, have students draw a rectangle that is one unit larger in length (e.g., 3 units by 2 units) and have them guess and check the number of 1 unit by 1 unit squares will be inside. Have students compare and discuss their results to the smaller rectangle. Then, have students repeat the process for different-size rectangles. Discuss what they observe[1] (MP.1, MP.2, MP.3, MP.3, MP.4, MP.5, MP.6, MP.7, MP.8). It is vital that students understand different representations of fair shares. Provide a collection of different-size circles and rectangles cut from paper. Ask students to fold some shapes into halves, some into thirds, and some into fourths. Have students then compare the locations of the folds in their shapes as a class and discuss the different representations for the fractional parts (MP.1, MP.2, MP.3, MP.4, MP.5, MP.6, MP.7, MP.8). To fold rectangles into thirds, ask students if they have ever seen how letters are folded to be placed in envelopes. Have them fold the paper very carefully to make sure the three parts are the same size. Ask them to discuss why the same process does not work to fold a circle into thirds (MP.1, MP.3, MP4, MP.5, MP. 6, MP.7).

Shape Manipulation and Spatial Reasoning

Instructional Strategies Continued:	**Instructional Strategies Continued:**	**Instructional Strategies Continued:**
Common Misconceptions: Some students may try and put smaller shapes together to make a larger version of the same shape but are not able to recognize the larger shape because of gaps and overlaps of the smaller shapes. For example, a student might be told to use four triangles to make a bigger triangle, but because of gaps and overlaps, the larger shape does not look like a triangle.  **With gaps and overlaps** **Without gaps and overlaps** Work with students so that they understand that when making new shapes, there cannot be any gaps or overlaps. Additionally, when working on making larger shapes out of smaller shapes, students may have a hard time understanding that shapes may have to be turned upside-down to make an enlarged version.	**Common Misconceptions:** Some students think that the size of the equal shares is directly related to the number of equal shares. So, they may think that fourths are larger than halves because there are four fourths and only two halves. Be sure to have students focus on the change in size of the fractional parts to the whole and not on the numerals within the fraction. Practice with physically dividing wholes into parts will help them to see that fourths are smaller than halves[1].	**Common Misconceptions:** Some students believe that a region model represents one out of two, three, or four fractional parts without regard to the fact that the parts have to be equal shares, e.g., dividing a circle with two equally spaced horizontal lines[1]. 
Assessment Procedure: **To be developed**	**Assessment Procedure:** **To be developed**	**Assessment Procedure:** **To be developed**
Vocabulary: Group/Join	**Vocabulary:** Half-circle quarter-circle Half Fourth	**Vocabulary:** Thirds Halves

Sample of Works Consulted

[1]Ohio Department of Education Model Curricula: http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?Page=3&TopicRelationID=1704&Content=136600

[2]The Common Core Writing Team. (29 May 2011). Progressions for the Common Core State Standards in Mathematics (draft): *K, Counting and Cardinality; K-5, Operations and Algebraic Thinking. http://commoncoretools.me/category/progressions/.*

[3]The Common Core Writing Team. (7 April 2011). Progressions for the Common Core State Standards in Mathematics (draft): *K-5, Number and Operations in Base Ten. http://commoncoretools.me/category/progressions/.*

[4]The Common Core Writing Team. (20 June 2011). Progressions for the Common Core State Standards in Mathematics (draft): *K-3, Categorical Data; Grades 2-5, Measurement Data. http://commoncoretools.me/category/progressions/.*

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